Efficient structure for single-bit digital comb filters and resonators

Sadik, Amin; Hussain, Zahir; O'Shea, Peter

https://researchrepository.rmit.edu.au/esploro/outputs/9921858524601341/filesAndLinks?institution=61RMIT_INST&index=null

Sadik, A., Hussain, Z., & O'Shea, P. (2005). Efficient structure for single-bit digital comb filters and resonators. Proceedings of the TENCON 2005 IEEE Region 10 Conference.
<https://doi.org/10.1109/TENCON.2005.301221>

Published Version: <https://doi.org/10.1109/TENCON.2005.301221>

Repository homepage: <https://researchrepository.rmit.edu.au>

© 2005 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

Downloaded On 2023/04/26 23:50:29 +1000

Efficient Structure for Single-Bit Digital Comb Filters and Resonators

Amin Z. Sadik

School of Engineering Systems
Queensland University of Technology
Currently a Visiting Researcher at SECE
RMIT, Melbourne, Australia
E-mail: amin.sadik@rmit.edu.au

Zahir M. Hussain, SMIEEE

School of Electrical & Computer Engineering
RMIT, Melbourne, Australia
Email: zmhussain@ieee.org

Peter O'Shea

School of Engineering Systems
Queensland University of Technology
Brisbane, Australia
pj.oshea@qut.edu.au

Abstract—A design technique for a single-bit digital comb filter is presented. The proposed filter response and performance are assessed in terms of signal-to-quantization-noise ratio (SQNR) and stability. It is found that the comb filter possesses a distinct frequency response in broadband signal applications. The same technique is utilized to design and simulate a single-bit N-period digital resonator. Feedback loop filters can be used to tune the frequency response of the $\Sigma\Delta$ modulators. The proposed design technique is efficient in hardware implementation.

I. INTRODUCTION

Single-bit processing has been attracting interest due to the promise of efficient and simple implementation [1]. Sigma-delta modulators ($\Sigma\Delta$'s) are the main single-bit modulators or analog-to-digital converters (ADC's). However, a drawback of the $\Sigma\Delta$ system is that high resolution can only be obtained for low to medium bandwidths [3]. This is so because the oversampling ratio (OSR) should be high for better resolution (might be several orders of magnitude higher than the Nyquist rate). Hence, It is difficult to handle broadband applications, such as broadband power-line communication (BPC), using an ordinary $\Sigma\Delta$.

The signal-to-quantization-noise (SQNR) can be improved by either increasing the order of the $\Sigma\Delta$ or increasing the OSR. Therefore, one approach to alleviate the low bandwidth bottleneck is to use a higher order $\Sigma\Delta$ with a reduced OSR. However, this will introduce the problem of unpredictable instability that is seemingly inherent in such high-order $\Sigma\Delta$ systems [2].

When designing a $\Sigma\Delta$, the noise-transfer function (NTF) should be given significant consideration. In general, it is designed as a high-pass function so that the quantization noise can be moved to higher frequency bands. In general, NTF can be classified into two classes:

- 1) Pure differentiation of order M : In this case the noise-shaping function (noise transfer function) can be expressed as follows:

$$N(z) = (1 - z^{-1})^M. \quad (1)$$

As the order M increases, more noise power will move to high frequency bands, hence, noise in the low frequency bands will reduce and, consequently, SNQR in the baseband is increased. Moreover, SQNR can be improved by increasing OSR. Due to the characteristics of this type of

Fig. 1. Block diagram of a second-order $\Sigma\Delta$.

NTF, its usage is usually limited in mid and low bandwidth applications such as audio applications.

- 2) non-monotonic transfer function: Pure differentiation response can be modified by introducing poles into NTF as follows:

$$N(z) = \frac{(z - 1)^M}{D(z)}. \quad (2)$$

In this case the NTF is an M^{th} -order polynomial with a leading coefficient of 1. It is simply a high-pass function, where the coefficients of the $\Sigma\Delta$ can be designed using analog filter techniques.

II. THEORY AND DESIGN

The design of single-bit $\Sigma\Delta$ is a non-trivial task. Many works in the literature have reported methods to estimate the performance of $\Sigma\Delta$ analytically [4]. However, these methods only approximate the actual behavior of $\Sigma\Delta$.

To characterize the modulator it is common to look at the STF and NTF. The STF describes how the modulator alters the original input signal spectrum. Ideally, STF is unity. In a similar manner the NTF describes how the modulator shapes noise away from the center frequency, f_c . For a low-pass modulator, $f_c = 0$ Hz (DC), and for a band-pass modulator f_c is often equal to $f_s/4$ for simpler design.

The NTF is the main design task which determines the amount of baseband noise shaping performed by the modulator.

A. Background

Fig.(1) shows a second-order $\Sigma\Delta M$ which contains two M^{th} -order FIR filters (can be assumed of different order as well) in its feedback loop to tune its response. Based on the linear model of $\Sigma\Delta M$, the z-transfer function of the above system can be found as follows:

$$Y(z) = \frac{X(z) + \frac{Q(z)}{H_1(z)H_2(z)}}{D(z)} \quad (3)$$

where $D(z)$ is given by:

$$D(z) = A(z) + B(z) \frac{1}{H_1(z)} + \frac{1}{H_1(z)H_2(z)} \quad (4)$$

with $A(z)$ and $B(z)$ being the transfer functions of the FIR filters (whose coefficients are $\{a_i | i = 0, \dots, M\}$ and $\{b_i | i = 0, \dots, M\}$ as follows:

$$A(z) = \sum_{i=0}^M a_i z^{-i} \quad (5)$$

$$B(z) = \sum_{i=0}^M b_i z^{-i}. \quad (6)$$

From above the signal and noise transfer functions can be expressed respectively as follows:

$$S(z) = \frac{1}{D(z)} \quad (7)$$

$$N(z) = \frac{1}{H_1(z)H_2(z)D(z)}. \quad (8)$$

Now, depending on the corresponding topology required for the standard second-order $\Sigma\Delta M$, the transfer functions $H_1(z)$ and $H_2(z)$ can take any of the following forms:

$$H_1(z) = H_2(z) = \frac{z^{-1}}{1 - z^{-1}} \quad (9)$$

$$H_1(z) = H_2(z) = \frac{1}{1 - z^{-1}} \quad (10)$$

$$H_1(z) = \frac{1}{1 - z^{-1}}; \quad H_2(z) = \frac{z^{-1}}{1 - z^{-1}}. \quad (11)$$

These topologies were found to have identical characteristics regarding noise shaping [5]. The only difference among them is the delay factor (z^{-1}) and the scaling gain. For instance, if we adopt the first form, $D(z)$ will be given as follows [6]:

$$D(z) = 1 + (a_o - 2)z^{-1} + (1 - b_o + b_1 + a_o)z^{-2} + G(z) \quad (12)$$

where $G(z)$ is given by:

$$G(z) = \sum_{i=1}^{M-1} b_{i+1} z^{-i-2} + \sum_{i=1}^M (a_i - b_i) z^{-i-2}. \quad (13)$$

For $a_o = 1$ and $b_o = 2$, the structure will be reduced to the standard $\Sigma\Delta$ topology, i.e., $D(z) = 1$, which implies two poles at $z = 0$. For coefficient values other than $a_o = 1$ and $b_o = 2$, $D(z)$ will be a second-order polynomial in z^{-1} , providing M equations with $2M$ unknown coefficients. These coefficients can

be found using different approaches [7]. However, $D(z)$ will not increase the order of noise shaping in the transfer function of the $\Sigma\Delta M$, but it may improve the stability of the system if it is well-designed.

B. The Proposed Structure

To improve system performance, $D(z)$ should be designed as an FIR low-pass filter to reduce the height of voltage steps at the output of the integrators [6], i.e., the input signal to $D(z)$ should not be attenuated at low frequency (when $z \rightarrow 1$). If we assume $D(z)$ as an $(M+2)^{\text{nd}}$ -order FIR filter with coefficients $\{d_i | i = 0, \dots, M+2\}$ as follows:

$$D(z) = \sum_{i=0}^{M+2} d_i z^{-i}, \quad (14)$$

Then, (in a semi-digital implementation, where the coefficients are implemented by analog means) a comb filter can be produced if the coefficients are equal, i.e., $d_0 = d_1 = \dots = d_{M+2} = \frac{1}{M+2}$. This means that we impose the following condition on the coefficients of $D(z)$:

$$\sum_{i=0}^{M+2} d_i = 1. \quad (15)$$

In this paper, our main intention is to design a single-bit digital comb filter for the purpose of efficient hardware implementation.

Now, the next step is to select proper functions to represent $H_1(z)$ and $H_2(z)$. As the z-transfer function of the comb filter is basically composed of equally spaced zeros around the unit circle circumference, then referring to eqn.3 we should choose $H_1(z)$ and $H_2(z)$ such that they match the required frequency response of the NTF as follows:

$$H_1(z) = H_2(z) = \frac{1}{(1 - z^{-M})}. \quad (16)$$

We choose $H_1(z)$ and $H_2(z)$ to have the same transfer function for convenience but not necessarily. Then, the output of the $\Sigma\Delta M$ will be as follows:

$$Y(z) = \frac{X(z) + Q(z)(1 - z^{-M})^2}{D(z)} \quad (17)$$

where $D(z)$ is now given by:

$$D(z) = A(z) + B(z)(1 - z^{-M}) + (1 - z^{-M})^2 \quad (18)$$

with $A(z)$ and $B(z)$ are as defined earlier. For simplicity, we propose $A(z) = B(z)$, but of course this is not the case always, it depends on the application to be achieved. Therefore, if $\sum_{i=0}^{M+2} d_i = 1$ as we proposed earlier, then after a few arithmetical manipulations we can find $A(z)$ as follows:

$$A(z) = B(z) = z^{-M}. \quad (19)$$

This implies that in this case both $\sum_{i=0}^M a_i = 1$ and $\sum_{i=0}^M b_i = 1$ and these functions are represented by a pure M -delay line.

Now, we re-design $D(z)$ such that it introduces some poles into the noise transfer function to comply with the non-monotonic noise transfer function type mentioned above (item-2. This can

Fig. 2. Block diagram of the proposed single-bit digital comb filter.

be carried out simply by putting $A(z) = B(z) = 1$ as is the case with a standard $\Sigma\Delta M$, where $D(z)$ will be given by:

$$D(z) = 3 - z^{-M} + z^{-2M}. \quad (20)$$

In this case $D(z)$ will introduce $2M$ poles into the NTF. These poles are distributed uniformly as conjugate pairs around a circle inside the unit circle on the z -plane. The radius r of this circle is $r = 0.577$. Therefore, it is expected that $D(z)$ will contribute to shaping the noise as well as to changing the STF in such away that converts the frequency response into M -period resonator, as will be seen next.

The designed single-bit digital comb filter is shown in Fig.(2). While the structure of the designed M -period resonator is shown in Fig.(3)

Fig.(4) shows the signal and noise frequency transfer functions $S(e^{j2\pi v})$ and $N(e^{j2\pi v})$ obtained from equation (20) with $M = 10$ as compared to those obtained from equation (18) with $M = 10$ which can be seen at Fig.(5). From these two figures we can expect the role that the NTF can play in tuning the $\Sigma\Delta$ system response. This may suggest introducing ternary filters in the feedback loop of the $\Sigma\Delta$ modulator.

C. Stability of the Proposed Structure

The stability of the system is decided by the poles in its transfer function. The single-bit comb filter does not contain prominent poles since $D(z) = 1$ implies two trivial poles at the center. Moreover, all zeros lies on the unit circle. On the other hand, the M -period resonator possesses $2M$ poles and all these poles are located inside the unit circle, in addition to the same zeros as in the NTF of the comb filter.

Fig. 3. Block diagram of the designed M -period resonator.

Fig. 4. Noise transfer function, $N(v)$ and signal transfer function, $S(v)$ for the designed single-bit comb filter

Fig. 5. Noise transfer function, $N(v)$ and signal transfer function, $S(v)$ for the designed single-bit M -period resonator

The stability of the modulator is assessed by looking at the quantizer input $x_q(n)$ Knee plot [10]. These were used to find which input values would result in the divergence of the quantizer input towards infinity. From which we may expect that this $\Sigma\Delta$ comb filter is to be stable as long as the input signal amplitude is limited by $|x_q| < 2$. Fig.(9) reveals this situation.

III. SIMULATION AND DISCUSSION

To verify the above analytic results, the proposed single-bit comb filter and multi-period resonator are simulated using MATLAB. The frequency response of the comb filter for $M = 10$ and OSR $R = 64$ is shown in Fig.(6).

Fig.(7) depicts the frequency response of the M -period single-bit resonator for $M = 10$ and OSR $R = 64$.

The signal-to-quantization-noise ratio (SQNR) is an essential performance measure for $\Sigma\Delta M$. The in-band SQNR is given in [8] as follows:

$$\text{SQNR}_{\text{in-band}} = \frac{2 \int_0^{0.5} |X(e^{j2\pi v})|^2 dv}{\int_{-1/(2R)}^{1/(2R)} |N(e^{j2\pi v})|^2 dv} \quad (21)$$

where $X(e^{j2\pi v})$ is the Fortier transform of the (oversampled) input signal $x(i)$. The SQNR can be estimated empirically. To do this, first the input signal spectrum must be removed from the output and replaced by interpolating the end points, and second the actual noise transfer function should be found to evaluate the SQNR as given by the expression above using

Fig. 6. Frequency Response of the proposed single-bit digital comb filter with order $M = 10$ and OSR $R = 64$.

Fig. 7. Frequency Response of the proposed single-bit M-period resonator filter order $M = 10$ and OSR $R = 64$.

Hannings-windowed FFT's. Sinusoidal inputs are used in this test. The input signal spectrum is chosen such that its spectral energy lies within a single FFT bin. Fig.(8) shows the simulated SNR as a function of OSR for sinusoidal inputs with different frequencies. As expected [9], doubling the sampling frequency reduces the noise power by about 9 dB, of which 3 dB is due to the reduction in power spectral density of the quantization noise, with additional 6 dB due to the action of the NTF.

It is also evident that, for the same value of OSR, there is an improvement of about 10 dB when the frequency of the input signal f_o is increased by an order of magnitude. This shows that this $\Sigma\Delta$ M topology lends itself well to broadband frequency applications, such as Broad-Band Power-line Communication

Fig. 8. SQNR against OSR for different input signal bandwidths with signal amplitude of 0.5.

Fig. 9. SQNR of the proposed single-bit comb filter versus the input amplitude for different values of OSR.

(BPL). This also suggests that the proposed single-bit $\Sigma\Delta$ comb filter can be utilized with relatively low OSR if the input frequency is high. In [11], we noticed that a similar structure has been proposed for UWB-OFDM applications.

Fig.(9) shows the SQNR as a function to the amplitude of the input signal for different OSRs. It can be seen clearly that the SQNR collapses at an absolute input levels less than 0.3dB.

IV. CONCLUSIONS

In this paper we propose a design technique for single-bit systems using a feedback path filter to tune the response of the $\Sigma\Delta$ modulator. This may suggest utilizing ternary filters in the feedback loop in future work. A single-bit digital comb filter is designed and its performance is evaluated in terms of signal-to-quantization noise ratio (SQNR), the dynamic range (input signal level), and stability. Moreover, we showed that the same design technique can be used for other single-bit systems, where we used it to design a multi-period resonator. It was shown that the proposed filters lend themselves very well to broadband input signals and can be utilized in emerging technologies such as the Broad-Band Power-line Communication (BPL).

ACKNOWLEDGMENT

This work is supported by the Australian Research Council under the ARC Discovery Grant DP0557429.

REFERENCES

- [1] A. C. Thompson, Z. M. Hussain, and P. O'Shea, "Efficient digital single-bit resonator," *IEE Electronics Letters*, vol. 40, no. 2, Feb. 2000.
- [2] HongMo Wang, "On the stability of third-order sigma-delta modulator," *IEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 47, no. 2, Oct. 2004.
- [3] J. Shiun, T. Hung, and P. Chu, "Novel noise shaping of cascaded sigma-delta modulator for wide bandwidth applications," *The 8th IEEE International conference on Electronics, Circuits and Systems, 2001 (ICECS 2001)*, vol. 3, pp. 1379-1382, Sept. 2001.
- [4] R. Gray, "Oversampled sigma-delta modulation," *IEEE Trans. Communications*, vol. COM-35, no. 5, pp. 481-489, May 1987.
- [5] S. R. Norsworthy, I. G. Post, and H. Scott Fetterman, "A 14-bit 80-kHz sigma-delta A/D converter: Modeling, Design, and Performance evaluation," *IEEE Journal of Solid-State Circuits*, vol. 24, no. 2, pp. 256-2266, April 1989.
- [6] O. Oliaei, "Sigma-Delta modulator with spectrally shaped feedback," *IEEE Transaction on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 50, no. 9, pp. 518-530, Sep. 2003.
- [7] D. K. Su and B. A. Wooley, "A CMOS oversampling D/A convertor with a current-mode semi-digital reconstruction filter," *IEEE J. of Solid State Circuits*, vol. 28, pp. 1224-1233, Dec. 1993.

- [8] M. D. Giles, I. Kale, and R. C. S. Morling, "Effect of coefficient quantization on a general class of sigma-delta based converters- a comparative study," *IEEE Conference on Advanced A/D and D/A Conversion Techniques and Their Applications*, , No. 466, pp. 106-110, 27-28 July 1999.
- [9] C. Dick and F. Harris, "FPGA signal processing using Sigma-Delta Modulation innovative combinations of techniques and hardware for system designer," *IEEE Signal Processing Magazine*, pp. 20-35, Jan. 2000.
- [10] R. Schreier, "An imperical study of high-order single-bit delta-segma modulators" *IEEE Transactionson on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 40, no. 8, August 1993.
- [11] K. Chuan, G. E. Sobelman, E. Saberinia, and A. H. Tewfik, "Performance of N-tone sigma-delta modulation for UWB-OFDM," *IEEE Communications Society*, pp. 1483-2486, 2004.