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Efficient Computation of Iceberg Cubes by Bounding Aggregate Functions

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Abstract—The iceberg cubing problem is to compute the multidimensional group-by partitions that satisfy given aggregation constraints. Pruning unproductive computation for iceberg cubing when nonantimonotone constraints are present is a great challenge because the aggregate functions do not increase or decrease monotonically along the subset relationship between partitions. In this paper, we propose a novel *bound prune cubing* (BP-Cubing) approach for iceberg cubing with nonantimonotone aggregation constraints. Given a cube over n dimensions, an aggregate for any group-by partition can be computed from aggregates for the most specific n -dimensional partitions (MSPs). The largest and smallest aggregate values computed this way become the bounds for all partitions in the cube. We provide efficient methods to compute tight bounds for base aggregate functions and, more interestingly, arithmetic expressions thereof, from bounds of aggregates over the MSPs. Our methods produce tighter bounds than those obtained by previous approaches. We present iceberg cubing algorithms that combine bounding with efficient aggregation strategies. Our experiments on real-world and artificial benchmark data sets demonstrate that BP-Cubing algorithms achieve more effective pruning and are several times faster than state-of-the-art iceberg cubing algorithms and that BP-Cubing achieves the best performance with the top-down cubing approach.

Index Terms—Data mining, data cube, pruning, data warehouses.

1 INTRODUCTION

WITH the multidimensional model for data warehouses, a data set consists of tuples over dimensions and measures, and queries involve aggregating the measures over partitions of tuples sharing identical dimension values. For example, the Structured Query Language (SQL) in Fig. 1a gives “the monthly total amount of sales for each city” on the Sales data set in Table 1. The (Month, City) group-by divides the data set into partitions of tuples sharing identical (Month, City) values, and the measure Sale is aggregated to yield Sum(Sale) for each partition. The Cube operator was proposed [6] to compute all potential group-bys of a data set, leading to the notion of a data cube.

The iceberg cube was proposed in [4], where only partitions whose aggregate values satisfy an aggregation constraint are produced. By adding “HAVING Count(*) ≥ 10 ” to the SQL query in Fig. 1a, we get an SQL query (in Fig. 1b) for computing those (Month, City) partitions, each having ≥ 10 tuples. The aggregation constraint Count(*) ≥ 10 is antimonotone: If a partition does not contain ≥ 10 tuples and thus fails the constraint, then all its subpartitions will have fewer tuples and also fail the constraint. The bottom-up cubing (BUC) strategy was proposed [4], where aggregates are computed starting from

the partition of all tuples and then followed recursively with the subpartitions, and antimonotone constraints are used for pruning.

In On-Line Analytical Processing (OLAP) applications, aggregation constraints often involve complex aggregates. The constraint Sum(x) $\geq 5,000$ and Var(x) ≤ 100 , which states that “the total profit (x) is at least \$5,000, and the variance of profit is at most \$100,” is a typical example of constraints for supporting enterprise data analysis and decision making. As x can be positive or negative, the subset relationship between a group¹ and its subgroups may not imply monotonically increasing (or decreasing) aggregate values. Computing iceberg cubes with such constraints has been a challenging problem. Existing proposals in the literature have all followed the approach of deriving weaker but antimonotone constraints [7], [15]. They are restricted in two aspects: 1) the weaker constraints derived from nonantimonotone constraints are often very loose and not effective enough for pruning, especially for complex constraints, and 2) aggregates are computed following the BUC framework, and only one group is aggregated at a time in the recursive partitioning process, which limits the amount of information readily available for pruning.

In this paper, we propose a novel technique, called *bound prune cubing* (BP-Cubing), for efficiently computing iceberg cubes with nonantimonotone aggregation constraints. The main ideas are to effectively derive tight bounds for possible aggregates and to use such bounds to prune unproductive computation. An important part is played by the “most specific partitions” (MSPs), namely, those nonempty partitions that cannot be further divided (for the subcube under consideration). MSPs can be viewed as the

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1. We use *group* and *partition* as synonyms.

SELECT Month, City, Sum(Sale)
 FROM Sales
 GROUP BY Month, City
 (a)

SELECT Month, City, Sum(Sale)
 FROM Sales
 GROUP BY Month, City
 HAVING Count(*) ≥ 10
 (b)

Fig. 1. Two SQL group-by queries on the sales data set. (a) An SQL group-by. (b) Adding a constraint to (a).

TABLE 1
A Sales Data Set, Partially Aggregated

Month	Product	SalesMan	City	Sum(Sale)	Count(*)
January	Toy	John	Perth	200	5
March	TV	Peter	Perth	100	40
March	TV	John	Perth	100	20
March	TV	John	Sydney	100	10
April	TV	Peter	Perth	100	8
April	Toy	Peter	Sydney	100	5

Month, Product, SalesMan, and City are dimensions. Sale is the measure.

basic units for computing data cubes, since other partitions are unions of some MSPs. The aggregate for any partition in the data cube can be computed from aggregating some MSPs. Decomposing an aggregate function into the base aggregate functions Count, Max, Min, and Sum on MSPs, the aggregate function can be tightly bounded. By using MSPs, nonexistent groupings of tuples will not influence the estimation of the bounds of aggregates,² which is another reason why tight bounds can be obtained.

With a four-dimensional data cube on the average Sale ($\text{Avg}(\text{Sale})$) of Sales, six MSPs are shown in Table 1. For each MSP, $\text{Avg}(\text{Sale}) = \text{Sum}(\text{Sale}) / \text{Count}(*).$ ³ We now illustrate cases where $\text{Sum}(\text{Sale}) > 0$ for each MSP (the general case is treated later). Both $\text{Sum}(\text{Sale})$ and $\text{Count}(*)$ increase monotonically over unions of “positive” MSPs. The upper bound of $\text{Avg}(\text{Sale})$ can be obtained from the upper bound for $\text{Sum}(\text{Sale})$ and the lower bound for $\text{Count}(*)$, which is $\text{Sum}_i(\text{Sum}(\text{Sale})) / \text{Min}_i(\text{Count}(*))$, where i ranges over the six MSPs. This can, in fact, be improved: The upper bound for $\text{Avg}(\text{Sale})$ is actually $\text{Max}_i \text{Avg}(\text{Sale})$, where i ranges over all MSPs (see Section 6). For Table 1, $\text{Max}_i \text{Avg}(\text{Sale})$ is $200/5 = 40$, reached at the group (January, Toy, John, Perth).

The MSPs can be obtained by a single scan of the raw data. The bound-prune technique described above will be applied recursively for all subcubes. We will optimize the computation for subcubes by reusing computed results for supercubes.

Our bound-prune technique can be applied in the BUC approach. More importantly, it also fits nicely into the top-down cubing approach, where multiway shared computation of aggregates improves cubing efficiency. Bound-prune is a general technique that can prune for constraints of the form “ $F(x) \geq \alpha$,” “ $F(x) \leq \alpha$,” or “ $F(x)$ in $[\alpha_1, \alpha_2]$.” The aggregate function F can be an SQL aggregate function or a commonly seen complex aggregate function such as Average (Avg) or Variance (Var). As will be shown later

2. This is an issue for other approaches.

3. For ease of discussion, we assume that NULL is not allowed for measures and, therefore, $\text{Count}(\text{Sale}) = \text{Count}(*).$

Fig. 2. Data cube lattice and subdata cube. (a) Cube (ABCD) decomposition. (b) Cube(BCD)|a1.

in our experiments on real-world and synthetic benchmark data sets, our BP-Cubing algorithms are several times faster than existing pruning techniques, including the most recent Divide-and-Approximate (DnA) algorithm [15].

Organizationally, Section 2 gives some preliminaries. Section 3 defines boundability of aggregate functions. We discuss how we can derive the bounds for base aggregates, their functions, and complex aggregates in Sections 4, 5, and 6, respectively. Section 7 then presents the BP-Cubing algorithms. We report experimental evaluations in Section 8. We discuss related works in Section 9 and conclude in Section 10.

2 SOME PRELIMINARIES ON DATA CUBES

We will use uppercase letters to denote dimensions and lowercase letters to denote dimension values. A group-by is a tuple of dimensions of the form (A, B, C) , and a partition of (A, B, C) is a tuple of dimension values of the form (a, b, c) .

The group-bys in a data cube form a lattice structure called the *cube lattice*. Fig. 2 shows an example four-dimensional cube lattice. The empty group-by (which aggregates all tuples in a data set) is at the bottom of the lattice, and the group-by with all dimensions is at the top. The edges in the lattice represent subset-superset relationships between group-bys.

A special value “*” is used in specifying partitions, with the meaning that it can match any value (of the applicable dimension). For the data set in Table 1, $(\text{Jan}, *, *, *)$ denotes the partition of tuples having Month = January and having no restriction on the other dimensions. For brevity, “*” is usually omitted in naming partitions; for example, $(\text{Jan}, *, *, *)$ is written as (Jan).

The subset relationship also exists between partitions from different group-bys. For example, the 3D partition (Jan, TV, Perth) is a subset of each of the 2D partitions (Jan, TV), (Jan, Perth), and (TV, Perth).

Given a data set S of n dimensions $A_1, \dots, A_n$ and a measure X , the corresponding data cube is usually referred to as $\text{Cube}(A_1, \dots, A_n)$. Here, we think of the cube as the set of possible group-bys or the set of possible groups for all group-bys. For example, the four-dimensional cube in Table 1 can be denoted as $\text{Cube}(\text{Month}, \text{Product}, \text{SalesMan}, \text{City})$.

For ease of discussion, we also refer to the cube as $\text{Cube}(X)$, and think of it as the “measure partitions” (the possible bags of measure values for the possible groups).

Specifically, let $g_1, \dots, g_m$ be all possible groups of the cube. By an abuse of notation, we will use X_i to denote the bag (multiset) $\{t[X] | t \in g_i\}$, and we will refer to $X_1, \dots, X_m$ as the *measure partitions*. Now, given an aggregate function F , we apply F to the measure partitions in the data cube to get $F(\text{Cube}(X)) = \{F(X_i) | X_i \in \text{Cube}(X)\}$.⁴ With the aggregate function $\text{Sum}(\text{Sale})$, we can write $\text{Sum}(\text{Cube}(\text{Sale}))$ for the four-dimensional cube in Table 1.

Aggregate functions are categorized into distributive, algebraic, and holistic functions, depending on how an aggregate on a partition can be computed from aggregates on its subpartitions [6]. 1) An aggregate function F is *distributive* if there is a function G such that $F(S) = G(\{F(S_i) | i = 1, \dots, n\})$ for a data set S and partitioning $\{S_1, \dots, S_n\}$ of S .⁵ For example, Sum and Max are distributive, with $G = F$, and Count is distributive, with $G = \text{Sum}$. 2) An aggregate function F is *algebraic* if it can be computed by a function H with several arguments, each of which is obtained by applying a distributive aggregate function. Avg , Var , $\text{Standard-Deviation}$, and MaxN and MinN ⁶ are algebraic functions; for example, Avg is algebraic, since $\text{Avg}(S) = \text{Sum}(S)/\text{Count}(S)$, and both Sum and Count are distributive. All distributive aggregations are algebraic. 3) An aggregate function F is *holistic* if it is not distributive or algebraic. Rank , Mode , and Median are examples of holistic functions. We will call the distributive aggregations used for computing algebraic functions F (item 2 above) the *auxiliary aggregates*.

Existing cubing algorithms have made use of the properties of distributive and algebraic functions to compute supergroups from subgroups [1], [10], [11], [17]. No efficient cubing algorithms for holistic functions have been reported. It is worth noting that in most applications, aggregate functions are algebraic functions. In the discussions below, the term “aggregate functions” will refer to distributive and algebraic functions unless specified otherwise.

3 BOUNDING DATA CUBES

The basic idea of bounding is to estimate the upper and lower bounds for a cube from the smallest or the MSPs of the base table. In this section, we define the boundability concept for aggregate functions. In Sections 4 and 5, we will discuss how we can bound the base aggregate functions Count , Max , Min , and Sum and how we can bound algebraic functions given as arithmetic expressions of the base aggregate functions.

Definition 1 (most specific partition and data cube core).

Given a data set on dimensions $A_1, \dots, A_n$ and measure X , all n -dimensional partitions form the core of the data cube:

$$\{(a_1, \dots, a_n) | a_i \in \text{domain}(A_i), a_i \neq \text{"*"}, 1 \leq i \leq n\}.$$

Each partition in the core is an MSP. The multiset of measure values for tuples in an MSP g , namely, $\{t[X] | t \in g\}$, is an MSP of the measure.

All aggregates in a data cube can be computed from its MSPs. For the Sales data set in Table 1 and $\text{Cube}(\text{Month},$

$\text{Product}, \text{SalesMan}, \text{City})$, the core consists of six $(\text{Month}, \text{Product}, \text{SalesMan}, \text{City})$ partitions, namely, $(\text{Jan}, \text{Toy}, \text{John}, \text{Perth})$, $(\text{Mar}, \text{TV}, \text{Peter}, \text{Perth})$, $(\text{Mar}, \text{TV}, \text{John}, \text{Perth})$, $(\text{Mar}, \text{TV}, \text{John}, \text{Sydney})$, $(\text{Apr}, \text{TV}, \text{Peter}, \text{Perth})$, and $(\text{Apr}, \text{Toy}, \text{Peter}, \text{Sydney})$. Any aggregate in this cube can be computed from a subset of the six MSPs. We thus can use MSPs to bound the aggregates in a data cube.

Definition 2 (Aggregate function bound). An upper bound of an aggregate function F for the data cube on X is a real number U such that for any partition X_i of $\text{Cube}(X)$, it is the case that $F(X_i) \leq U$. Similarly, we can define a lower bound for $F(\text{Cube}(X))$ to be a real number L such that for any partition X_i of $\text{Cube}(X)$, $F(X_i) \geq L$.

Observation 1. Given a data cube on measure X and an aggregate function F , the tightest upper bound and lower bound are, respectively, reached by the largest and smallest aggregate values that can be produced by any set of MSPs of the data cube.

Example 1. In Table 1, $\text{Sum}(\text{Sale})$ for any group in the data cube is not larger than the sum of $\text{Sum}(\text{Sale})$ for all six MSPs, which is 700. On the other hand, $\text{Sum}(\text{Sale})$ for any group in the data cube is not smaller than the minimal $\text{Sum}(\text{Sale})$ among the six MSPs, which is 100. Therefore, for $\text{Sum}(\text{Sale})$, $\text{Cube}(\text{Month}, \text{Product}, \text{SalesMan}, \text{City})$ has the tightest upper bound of 700 and the tightest lower bound of 100.

The notion of MSP and data cube core is also applied to subdata cubes. Fig. 2a shows $\text{Cube}(\text{ABCD})$, and its MSPs are the ABCD partitions. The polygon on the left of Fig. 2a denotes a set of lattice structures, as shown in Fig. 2b. The polygon on the right of Fig. 2a represents $\text{Cube}(\text{BCD})$. The MSPs for $\text{Cube}(\text{BCD})$ are BCD partitions, which are unions of ABCD partitions, with equal values for B, C, and D. The lattice structure, as shown in Fig. 2b, is called a subdata cube. For the lattice in Fig. 2b, (a_1, B, C, D) partitions, which are a subset of the ABCD partitions, form its core. The relationship between cores for a cube and its subcubes, and that for a cube and its lower-dimensional counterparts, are important for computing bounds from MSPs, without incurring extra cost and for effective pruning with bounds, as shown in our BP-Cubing algorithms, discussed in Section 7. In later discussions, we use the term “data cube” to refer to a standard or subdata cube.

Definition 3 (Subdata cube). Consider dimensions $A_1, \dots, A_n$ and $B_1, \dots, B_k$ and values

$$b_1 \in \text{domain}(B_1), \dots, b_k \in \text{domain}(B_k).$$

The groups aggregating tuples that satisfy $B_i = b_i$ for $1 \leq i \leq k$ form a subdata cube, or simply a subcube, denoted as $\text{Cube}(A_1, \dots, A_n) | b_1, \dots, b_k$. The core of the subdata cube comprises MSPs with $B_i = b_i$ for $1 \leq i \leq k$. $B_1, \dots, B_k$ are conditional dimensions for the subcube.

Example 2. Continuing with Example 1, consider

$$\text{Cube}(\text{Product}, \text{SalesMan}, \text{City}) | \text{Mar}.$$

There are three MSPs for the data cube: $(\text{Mar}, \text{TV}, \text{Peter}, \text{Perth})$, $(\text{Mar}, \text{TV}, \text{John}, \text{Perth})$, and

4. Strictly speaking, $F(X_i)$ is the F aggregate of X for group g_i .

5. $S = \bigcup_{i=1}^n S_i$ and $S_i \cap S_j = \emptyset$ for $i \neq j$.

6. MaxN and MinN return, respectively, the N largest and smallest values.

(Mar, TV, John, Sydney). Tighter bounds can be obtained on subcubes. Indeed, the tightest upper bound for $\text{Sum}(\text{Sale})$ for the data cube is $100 + 100 + 100 = 300$, and the tightest lower bound is $\text{Min}(\{100, 100, 100\}) = 100$.

Based on Observation 1, we can see that a data cube can be bounded from its MSPs. However, exhaustively checking the power set of MSPs is equivalent to computing the complete data cube and is not computationally feasible. We define boundable aggregate functions as follows.

Definition 4 (Boundable aggregate function). An aggregate function F is boundable for a data cube if some upper and lower bounds of F for the data cube can be determined by some algorithm with a single scan of some auxiliary aggregate values of the MSPs of the data cube. We will use $\overline{F(\text{Cube}(X))}$ and $\underline{F(\text{Cube}(X))}$ to denote, respectively, the upper and lower bounds, computed by a given single-scan algorithm.⁷

It should be pointed out that the bounds computed may not be the tightest. To ensure the effectiveness of pruning, we aim to derive bounds that are as tight as possible. Let us use some example aggregate functions to explain this definition.

Example 3. Consider the aggregate function Count and measure X with n MSPs $X_1, \dots, X_n$, where

$$\text{Count}(X) = \text{Sum}(\{\text{Count}(X_i) | i = 1 \dots n\}).$$

The auxiliary aggregate function is Count . For any partition g of $\text{Cube}(X)$, the number of tuples in g is not larger than the total number of tuples of all MSPs; in other words, $\text{Count}(g) \leq \text{Sum}(\{\text{Count}(X_i) | i = 1 \dots n\})$. On the other hand, we also have

$$\text{Count}(g) \geq \text{Min}(\{\text{Count}(X_i) | i = 1 \dots n\}).$$

As a result, $\text{Sum}(\{\text{Count}(X_i) | i = 1 \dots n\})$ is an upper bound, and $\text{Min}(\{\text{Count}(X_i) | i = 1 \dots n\})$ is a lower bound. They can be obtained by a single scan of the auxiliary aggregates of MSPs. Therefore, $\text{Count}(\text{Cube}(X))$ is boundable.

We now give an example of aggregate functions whose bounds can be infinite.

Example 4. Given measure X and aggregate function $1/\text{Sum}$, consider $\text{Cube}(X)$ with MSPs $X_1, \dots, X_n$. Then, $1/\text{Sum}(X) = 1/\text{Sum}(\{\text{Sum}(X_i) | i = 1 \dots n\})$. As will be explained later in Tables 3 and 4, the signs of $\text{Sum}(X_i)$ are important for computing the bounds:

1. If $\text{Sum}(X_i) > 0$ for all $i = 1 \dots n$, then the upper and lower bounds are computed from, respectively, the positive lower and upper bounds of $\text{Sum}(\text{Cube}(X))$:

$$\overline{1/\text{Sum}(\text{Cube}(X))} = 1/\text{Min}(\{\text{Sum}(X_i) | i = 1 \dots n\})$$

7. We will write $\overline{F(\text{Cube}(X))}$ and $\underline{F(\text{Cube}(X))}$ to mean, respectively, the upper and lower bounds computed by our algorithm. That algorithm is the sum of the bounding formulas presented in this paper.

TABLE 2
The Bounds of Base Aggregate Functions

F	$\overline{F(\text{Cube}(X))}; \underline{F(\text{Cube}(X))}$
Count	$\text{Sum Count}(X_i); \text{Min Count}(X_i)$
Max	$\text{Max}_i \text{Max}(X_i); \text{Min}_i \text{Max}(X_i)$
Min	$\text{Max}_i \text{Min}(X_i); \text{Min}_i \text{Min}(X_i)$
Sum	$\begin{array}{l} \text{Sum Sum}(X_i) \text{ if (a)} \quad \text{Sum Sum}(X_i) \text{ if (b)} \\ \text{Sum}(X_i) > 0 \quad \quad \quad \text{Sum}(X_i) < 0 \\ \text{Max Sum}(X_i) \text{ otherwise} \quad \text{Min Sum}(X_i) \text{ otherwise} \end{array}$

Given a data set, X is the measure, and $X_1, \dots, X_n$ are the MSPs. (a) There is i such that $\text{Sum}(X_i) > 0$. (b) There is i such that $\text{Sum}(X_i) < 0$.

and

$$\overline{1/\text{Sum}(\text{Cube}(X))} = 1/\text{Sum}(\{\text{Sum}(X_i) | i = 1 \dots n\}).$$

2. If $\text{Sum}(X_i) < 0$ for all $i = 1 \dots n$, then the upper and lower bounds are computed from, respectively, the negative lower and upper bounds of $\text{Sum}(\text{Cube}(X))$ as

$$\overline{1/\text{Sum}(\text{Cube}(X))} = 1/\text{Sum}(\{\text{Sum}(X_i) | i = 1 \dots n\})$$

and

$$\underline{1/\text{Sum}(\text{Cube}(X))} = 1/\text{Max}(\{\text{Sum}(X_i) | i = 1 \dots n\}).$$

3. For general cases, where $\text{Sum}(X_i)$ can be positive or negative, the upper bound $\overline{1/\text{Sum}(\text{Cube}(X))}$ is computed from the smallest $\text{Sum}(x) \geq 0$ for any partition in the cube. For any group g of $\text{Cube}(X)$, $\text{Sum}(g)$ can be the sum of any positive or negative MSP aggregates and can have a minimum of 0. Thus, the upper bound for $1/\text{Sum}(\text{Cube}(X))$ is ∞ . Similarly, the lower bound $\underline{1/\text{Sum}(\text{Cube}(X))}$ is computed from the largest $\text{Sum}(x) < 0$ for any partition in the cube, which can be a negative approaching 0. Therefore, the lower bound for $1/\text{Sum}(\text{Cube}(X))$ is $-\infty$.

Computing the upper and lower bounds for arithmetic expressions of base aggregate functions such as $1/\text{Sum}$ is summarized later in Table 4.

4 BOUNDING BASE AGGREGATE FUNCTIONS

We now consider how we can bound the base aggregate functions of SQL.

Theorem 1. The base aggregate functions Count , Min , Max , and Sum are boundable, and their bounds are listed in Table 2.

We use the following example for Sum to explain the reasoning for the bounds of Table 2 and how they are obtained by a single scan of MSPs.

Example 5. Suppose we are to compute a data cube on measure X , which may take positive or negative values. Let the MSPs of X be $X_1, \dots, X_n$. With a single scan of $X_1, \dots, X_n$, we have $\text{Sum}(X_i) (1 \leq i \leq n)$ and the positive/negative and maximal/minimal values among them:

TABLE 3
The Sign Bounds of Base Aggregate Functions

F	$F^+(\text{Cube}(X)); F^+(\text{Cube}(X))$	$F^-(\text{Cube}(X)); F^-(\text{Cube}(X))$
Count	Sum Count(X_i); Min Count(X_i)	n/a ; n/a
Max	Max(X_i); Min(X_i)	Max(X_i); Min(X_i)
Min	Max(X_i); Min(X_i)	Max(X_i); Min(X_i)
Sum	Sum(X_i); if Sum(X_i) ≥ 0 for $i = 1..n$ Min Sum(X_i), otherwise 0.	If Sum(X_i) < 0 for $i = 1..n$ Max Sum(X_i), otherwise 0; Sum Sum(X_i).

X is the measure of a data set, and the MSPs for the data cube are $X_1, \dots, X_n$.

1. If there exists i such that $\text{Sum}(X_i) > 0$, then the sum for any group in $\text{Cube}(X)$ is not greater than the sum of all such positive $\text{Sum}(X_i)$. Otherwise, $\text{Sum}(X_i) \leq 0$ for $i = 1 \dots n$; the sum for any group is not greater than the maximal $\text{Sum}(X_i)$. Thus,

$$\overline{\text{Sum}(\text{Cube}(X))} = \sum_{\text{Sum}(X_i) > 0} \text{Sum}(X_i)$$

if there exists $\text{Sum}(X_i) > 0$;

$$\overline{\text{Sum}(\text{Cube}(X))} = \text{MaxSum}(X_i), \text{ otherwise.}$$

2. If there exists i such that $\text{Sum}(X_i) < 0$, then the sum of any group in $\text{Cube}(X)$ is not less than the sum of all such $\text{Sum}(X_i)$. Otherwise, $\text{Sum}(X_i) \geq 0$ for all $i = 1 \dots n$; the sum of any group is not less than the minimal $\text{Sum}(X_i)$. Therefore,

$$\underline{\text{Sum}(\text{Cube}(X))} = \sum_{\text{Sum}(X_i) < 0} \text{Sum}(X_i),$$

if there exists $\text{Sum}(X_i) < 0$;

$$\underline{\text{Sum}(\text{Cube}(X))} = \text{MinSum}(X_i), \text{ otherwise.}$$

To bound functions that are arithmetic expressions of base aggregate functions, sometimes, the sign bounds of aggregations need to be computed (see Section 5). These include positive upper bound ($\overline{F^+}$), positive lower bound ($\underline{F^+}$), negative upper bound ($\overline{F^-}$), and negative lower bound ($\underline{F^-}$), where F is either a base aggregate function Min, Max, Sum, or Count or an arithmetic expression of base aggregate functions. $\overline{F^+ \text{Cube}(X)}$ is defined to be a real $l \geq 0$ obtained by a single-scan algorithm such that $l \geq F(g) \geq 0$ for any partition g in $\text{Cube}(X)$. The other sign bounds can be defined similarly. The positive (negative) bounds are applicable only if there are nonnegative (negative) aggregates among the MSPs. The sign bounds of base aggregate functions are listed in Table 3.

We explain how the sign bounds for Sum are computed. Computation for other functions is straightforward. We discuss how we can compute the positive bounds of Sum, with discussions on the negative bounds mirror the same

TABLE 4
Bounding Arithmetic Expressions

op	bounds for the expression (E_1 op E_2)
+	$\overline{(E_1 + E_2)} = \overline{E_1} + \overline{E_2}, \underline{(E_1 + E_2)} = \underline{E_1} + \underline{E_2}$
-	$\overline{(E_1 - E_2)} = \overline{E_1} - \underline{E_2}, \underline{(E_1 - E_2)} = \underline{E_1} - \overline{E_2}$
×	$\overline{(E_1 \times E_2)} = \text{Max}(\overline{E_1^+ \times E_2^+}, \overline{E_1^- \times E_2^-}, \overline{E_1^+ \times E_2^-}, \overline{E_1^- \times E_2^+})$ $\underline{(E_1 \times E_2)} = \text{Min}(\underline{E_1^+ \times E_2^+}, \underline{E_1^- \times E_2^-}, \underline{E_1^+ \times E_2^-}, \underline{E_1^- \times E_2^+})$
/	$\overline{(E_1 / E_2)} = \text{Max}(\overline{E_1^+ / E_2^+}, \overline{E_1^- / E_2^-}, \overline{E_1^+ / E_2^-}, \overline{E_1^- / E_2^+})$ $\underline{(E_1 / E_2)} = \text{Min}(\underline{E_1^+ / E_2^+}, \underline{E_1^- / E_2^-}, \underline{E_1^+ / E_2^-}, \underline{E_1^- / E_2^+})$

The short notations E_1 , E_2 , and $(E_1 \text{ op } E_2)$ denote $E_1(\text{Cube}(X))$, $E_2(\text{Cube}(X))$, and $(E_1 \text{ op } E_2)(\text{Cube}(X))$, respectively, where X is the measure of a data set. When the operator is “ $\times$ ” or “/,” to bound $(E_1 \text{ op } E_2)$, not all four kinds of computation are applicable. As indicated, the signs of $E_1(X_i)$ and $E_2(X_i)$ ($i = 1 \dots n$) decide the applicable computation.

case as in positive bounds. The positive upper bound is the sum of all positive sums of the MSPs. To compute the positive lower bound, two cases exist: 1) if $\text{Sum}(X_i) \geq 0$ for all X_i ($1 \leq i \leq n$), then the positive lower bound is the minimal $\text{Sum}(X_i) \geq 0$ and 2) if the sums of MSPs include both negative and positive ones, then the sum of any set of MSPs can be as low as 0, and so, 0 is the positive lower bound.

5 BOUNDING ARITHMETIC EXPRESSIONS OF BASE AGGREGATE FUNCTIONS

Consider a data cube $\text{Cube}(X)$ and an arithmetic expression of auxiliary aggregates $E = E_1 \text{ op } E_2$, where E_1 and E_2 are base aggregate functions or arithmetic expressions of base aggregates. If the operator is “+” or “-,” then the upper (lower) bounds of $E(\text{Cube}(X))$ can be computed from the upper (lower) bounds of $E_1(\text{Cube}(X))$ and $E_2(\text{Cube}(X))$. If the operator is “ $\times$ ” or “/,” then we need to use the sign bounds of $E_1(\text{Cube}(X))$ and $E_2(\text{Cube}(X))$ to bound $E(\text{Cube}(X))$.

Proposition 1. Given $\text{Cube}(X)$ on measure X and an arithmetic expression $E = E_1 \text{ op } E_2$, where the operator is “+,” “-,” “ $\times$,” or “/,” and E_1 and E_2 are base aggregate functions or arithmetic expressions of base aggregates, $\overline{E(\text{Cube}(X))}$ and $\underline{E(\text{Cube}(X))}$ can be computed from the bounds or sign bounds of $E_1(\text{Cube}(X))$ and $E_2(\text{Cube}(X))$, as shown in Table 4.

Proof. Let $X_1, \dots, X_n$ be the MSPs of $\text{Cube}(X)$. We consider each of the four possible operators:

1. $E_1 + E_2$. By the definition of upper bound, for any group $g \in \text{Cube}(X)$, $E_1(g) \leq \overline{E_1(\text{Cube}(X))}$, and $E_2(g) \leq \overline{E_2(\text{Cube}(X))}$. We have

$$E_1(g) + E_2(g) \leq \overline{E_1(\text{Cube}(X))} + \overline{E_2(\text{Cube}(X))}.$$

Thus,

$$\overline{(E_1 + E_2)(\text{Cube}(X))} = \overline{E_1(\text{Cube}(X))} + \overline{E_2(\text{Cube}(X))}.$$

The proof is similar for

$$\underline{(E_1 + E_2)(\text{Cube}(X))} = \underline{E_1(\text{Cube}(X))} + \underline{E_2(\text{Cube}(X))}.$$

2. $E_1 - E_2$. Given $g \in \text{Cube}(X)$, $E_1(g) \leq \overline{E_1(\text{Cube}(X))}$ and $E_2(g) \geq \underline{E_2(\text{Cube}(X))}$. Thus,

$$E_1(g) - E_2(g) \leq \overline{E_1(\text{Cube}(X))} - \underline{E_2(\text{Cube}(X))}$$

and

$$\overline{(E_1 - E_2)(\text{Cube}(X))} = \overline{E_1(\text{Cube}(X))} - \underline{E_2(\text{Cube}(X))}.$$

The proof is similar for

$$\underline{(E_1 - E_2)(\text{Cube}(X))} = \underline{E_1(\text{Cube}(X))} - \overline{E_2(\text{Cube}(X))}.$$

3. $E_1 \times E_2$. Let g be a group of $\text{Cube}(X)$. Let $Y_1, Y_2, \dots, Y_k$ be the MSPs that are contained in g . Based on their signs under E_1 , we form two subsets from these MSPs:

$$\begin{aligned} PE_1 &= \{Y_j | E_1(Y_j) \geq 0, 1 \leq j \leq k\}, \\ NE_1 &= \{Y_j | E_1(Y_j) < 0, 1 \leq j \leq k\}. \end{aligned}$$

Similarly, based on their signs under E_2 , we form the following subsets from these MSPs:

$$\begin{aligned} PE_2 &= \{Y_j | E_2(Y_j) \geq 0, 1 \leq j \leq k\}, \text{ and} \\ NE_2 &= \{Y_j | E_2(Y_j) < 0, 1 \leq j \leq k\}. \end{aligned}$$

For the *upper bound*, we first consider the case where $PE_1 \neq \emptyset$ and $PE_2 \neq \emptyset$ or $NE_1 \neq \emptyset$ and $NE_2 \neq \emptyset$. $E_1(g) \times E_2(g)$ can have positive values. Thus,

$$E_1(g) \times E_2(g) \leq \text{Max} \left(\overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))}, \right. \\ \left. \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))} \right).$$

Otherwise, $E_1(g) \times E_2(g)$ can only be negative. We have

$$E_1(g) \times E_2(g) \leq \text{Max} \left(\overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))}, \right. \\ \left. \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))} \right).$$

For all cases, by combining the two inequalities with $E_1(g) \times E_2(g)$ on the left-hand side, we get $\overline{(E_1 \times E_2)(\text{Cube}(X))}$:

$$\begin{aligned} \text{Max} \left(\overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))}, \right. \\ \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))}, \\ \overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))}, \\ \left. \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))} \right). \end{aligned}$$

The proof for the *lower bound* is quite similar to that for the upper bound. The possible positive values of g are

$$\geq \text{Min} \left(\overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))}, \right. \\ \left. \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))} \right)$$

and the possible negative values of g are

$$\geq \text{Min} \left(\overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))}, \right. \\ \left. \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))} \right).$$

By combining the last two inequalities, we get $\underline{(E_1 \times E_2)(\text{Cube}(X))}$:

$$\begin{aligned} \text{Min} \left(\overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))}, \right. \\ \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))}, \\ \overline{E_1^+(\text{Cube}(X))} \times \overline{E_2^-(\text{Cube}(X))}, \\ \left. \overline{E_1^-(\text{Cube}(X))} \times \overline{E_2^+(\text{Cube}(X))} \right). \end{aligned}$$

4. E_1/E_2 . The proof is similar to that for the operator “ $\times$ ” and is omitted. $\square$

Example 6. Consider a data cube $\text{Cube}(X)$, where the measure X can be positive or negative, and the aggregate function Avg. Suppose the MSPs are $X_1, \dots, X_n$. For any MSP X_i , we note that $\text{Avg}(X_i) = \text{Sum}(X_i)/\text{Count}(X_i)$, and $\text{Count}(X_i)$ is always positive. Following Table 4, $\text{Avg}(\text{Cube}(X))$ is

$$\text{Max} \left(\overline{\text{Sum}^+(\text{Cube}(X))} / \overline{\text{Count}^+(\text{Cube}(X))}, \right. \\ \left. \overline{\text{Sum}^-(\text{Cube}(X))} / \overline{\text{Count}^+(\text{Cube}(X))} \right)$$

and $\underline{\text{Avg}(\text{Cube}(X))}$ is

$$\text{Min} \left(\overline{\text{Sum}^+(\text{Cube}(X))} / \overline{\text{Count}^+(\text{Cube}(X))}, \right. \\ \left. \overline{\text{Sum}^-(\text{Cube}(X))} / \overline{\text{Count}^+(\text{Cube}(X))} \right).$$

If there exists some i such that $\text{Sum}(X_i) > 0$, then

$$\overline{\text{Avg}(\text{Cube}(X))} = \overline{\text{Sum}^+(\text{Cube}(X))} / \overline{\text{Count}(\text{Cube}(X))}, \text{ and} \\ \underline{\text{Avg}(\text{Cube}(X))} = \underline{\text{Sum}^+(\text{Cube}(X))} / \underline{\text{Count}(\text{Cube}(X))}.$$

Otherwise, $\text{Sum}(X_i) \leq 0$ for all $i = 1 \dots n$; then, we have

$$\overline{\text{Avg}(\text{Cube}(X))} = \overline{\text{Sum}^-(\text{Cube}(X))} / \overline{\text{Count}(\text{Cube}(X))}, \text{ and} \\ \underline{\text{Avg}(\text{Cube}(X))} = \underline{\text{Sum}^-(\text{Cube}(X))} / \underline{\text{Count}(\text{Cube}(X))}.$$

We then follow Table 3 to compute the sign bounds in the above expressions.

The sign bounds for base aggregate functions are listed in Table 3. We now discuss how we can derive the sign bounds for arithmetic expressions. Note that in Table 5, positive (negative) bounds are applicable only if there exists an MSP X_i in the cube such that

$$(E_1(X_i) \text{ op } E_2(X_i)) \geq 0 \text{ (} (E_1(X_i) \text{ op } E_2(X_i)) < 0 \text{)}.$$

TABLE 5
The Sign Bounds of Arithmetic Expressions

op	$(E_1 \text{ op } E_2)^+; (E_1 \text{ op } E_2)^+$	$(E_1 \text{ op } E_2)^-; (E_1 \text{ op } E_2)^-$
+	$\overline{E_1 + E_2};$ if $E_1 + E_2 \geq 0$, $\underline{E_1 + E_2}$, otherwise 0.	if $E_1 + E_2 < 0$, $\overline{E_1 + E_2}$, otherwise 0; $\underline{E_1 + E_2}$
-	$\overline{E_1 - E_2};$ if $E_1 - E_2 \geq 0$, $\underline{E_1 - E_2}$, otherwise 0.	if $E_1 - E_2 < 0$, $\overline{E_1 - E_2}$, otherwise 0; $\underline{E_1 - E_2}$
×	$\text{Max}(\overline{E_1^+ \times E_2^+}, \underline{E_1^- \times E_2^-});$ $\text{Min}(\overline{E_1^+ \times E_2^+}, \underline{E_1^- \times E_2^-})$	$\text{Max}(\overline{E_1^+ \times E_2^-}, \underline{E_1^- \times E_2^+});$ $\text{Min}(\overline{E_1^+ \times E_2^-}, \underline{E_1^- \times E_2^+})$
/	$\text{Max}(\overline{E_1^+ / E_2^+}, \underline{E_1^- / E_2^-});$ $\text{Min}(\overline{E_1^+ / E_2^+}, \underline{E_1^- / E_2^-})$	$\text{Max}(\overline{E_1^+ / E_2^-}, \underline{E_1^- / E_2^+});$ $\text{Min}(\overline{E_1^+ / E_2^-}, \underline{E_1^- / E_2^+})$

Short notations E_1 , E_2 , and $(E_1 \text{ op } E_2)$ denote $E_1(\text{Cube}(X))$, $E_2(\text{Cube}(X))$, and $(E_1 \text{ op } E_2)(\text{Cube}(X))$, respectively, where X is the measure of a data set.

For example, the positive upper bound

$$\overline{(E_1 + E_2)(\text{Cube}(X))}^+$$

is applicable only if there exists X_i such that

$$E_1(X_i) + E_2(X_i) \geq 0.$$

For the operators “+” and “−”, when $E_1 + E_2$ or $E_1 - E_2$ can be positive or negative for MSPs, the positive lower and negative upper bounds for both expressions are 0: this is because the combination of positive and negative MSPs can produce the aggregate value of 0. For the operators “×” and “/”, the signs of E_1 and E_2 for MSPs decide the applicable computation. We use an example to explain Table 5.

Example 7. Given data cube $\text{Cube}(X)$ with MSPs $X_1, \dots, X_n$, consider the aggregate function

$$E(X) = 1/(\text{Max}(X) + \text{Min}(X)).$$

To compute $\overline{E(\text{Cube}(X))}$, three cases can arise:

Case 1. $\text{Min}(X_i) \geq 0$ for all $i = 1 \dots n$. It follows that $\text{Max}(X_i) \geq 0$ for all $i = 1 \dots n$. Then,

$$\overline{E(\text{Cube}(X))} = 1/(\overline{\text{Max}^+(\text{Cube}(X))} + \overline{\text{Min}^+(\text{Cube}(X))}).$$

Case 2. $\text{Max}(X_i) \leq 0$ and $\text{Min}(X_i) \leq 0$ for all $i = 1 \dots n$. Then,

$$\overline{E(\text{Cube}(X))} = 1/(\overline{\text{Max}^-(\text{Cube}(X))} + \overline{\text{Min}^-(\text{Cube}(X))}).$$

Case 3. $\text{Max}(X_i)$ and $\text{Min}(X_i)$ can be positive or negative ($1 \leq i \leq n$). Based on Table 4, we have $\overline{E(\text{Cube}(X))} = 1/(\overline{\text{Max}() + \text{Min}()})^+(\text{Cube}(X))$. Based on Table 5, $(\overline{\text{Max}() + \text{Min}()})^+(\text{Cube}(X))$ is 0, which means that the upper bound for $1/(\overline{\text{Max}() + \text{Min}()})^+(\text{Cube}(X))$ is ∞ .

Proposition 1 leads to the following theorem about bounding arithmetic expressions of base aggregate functions.

Theorem 2. Given a data cube on measure X , an arithmetic expression $E = E_1 \text{ op } E_2$ of base aggregate functions, where the operator is +, −, ×, or /. E is boundable for $\text{Cube}(X)$, and its bounds can be computed following Tables 2, 3, 4, and 5.

Proof. By Proposition 1, we can compute the bounds for an arithmetic expression from the bounds or sign bounds of the operand expressions, as listed in Table 4. Table 5 shows how the sign bounds of any expression can be computed from the bounds for base aggregate functions. Tables 2 and 3 demonstrate that a single scan of the MSPs of a data cube can compute the bounds of all base aggregate functions. As a result, all arithmetic expressions of base aggregate functions are boundable, and the bounds can be computed following Tables 2, 3, 4, and 5. $\square$

We now show how we can compute the upper bound for the complex aggregate function Var based on the theorem.

Example 8. The Var for $X = \{x_i | i = 1 \dots N\}$ is defined as⁸ $\text{Var}(X) = \frac{\sum_i (x_i - \bar{X})^2}{N}$, where $\bar{X}$ denotes the Avg of X . It can be rewritten as follows:

$$\begin{aligned} \text{Var}(X) &= \frac{\sum_i (x_i^2 - 2 \times \bar{X} \times x_i + \bar{X}^2)}{N}, \\ &= \frac{\sum_i x_i^2}{N} - \frac{2 \times \bar{X} \times \sum_i x_i}{N} + \frac{N \times \bar{X}^2}{N}, \\ &= \frac{\sum_i x_i^2}{N} - 2 \times \bar{X} + \bar{X}^2 = \frac{\sum_i x_i^2}{N} - \bar{X}^2, \\ &= \frac{\text{Sum}_i x_i^2}{\text{Count}(X)} - \left(\frac{\text{Sum}(X)}{\text{Count}(X)} \right)^2. \end{aligned}$$

Let $\text{QSum}(X)$ denote $\text{Sum}_i x_i^2$. It follows that $\text{Var}(X)$ is an arithmetic expression of $\text{QSum}(X)$, $\text{Sum}(X)$, and $\text{Count}(X)$. Consider a data cube $\text{Cube}(X)$ with MSPs $X_1, \dots, X_n$. Based on Table 4, we have

$$\begin{aligned} \overline{\text{Var}(\text{Cube}(X))} &= \overline{\left(\frac{\text{QSum}()}{\text{Count}()} \right)(\text{Cube}(X))} \\ &\quad - \overline{\left(\frac{\text{Sum}()}{\text{Count}()} \right)^2(\text{Cube}(X))}. \end{aligned}$$

We next consider each of the two subexpressions separately.

We first consider $\overline{\left(\frac{\text{QSum}()}{\text{Count}()} \right)(\text{Cube}(X))}$: For any MSP X_i , $\text{QSum}(X_i)$ and $\text{Count}(X_i)$ are always positive. By following Table 4, we have the following fractional expression, where the numerator and denominator expressions can be computed following Table 2: $\frac{\overline{\text{QSum}()}}{\overline{\text{Count}()}}(\text{Cube}(X)) = \frac{\overline{\text{QSum}(\text{Cube}(X))}}{\overline{\text{Count}(\text{Cube}(X))}}$.

We then consider $\overline{\left(\frac{\text{Sum}()}{\text{Count}()} \right)^2(\text{Cube}(X))}$. Based on Table 4, we have

$$\begin{aligned} \left(\frac{\overline{\text{Sum}()}}{\overline{\text{Count}()}} \right)^2 &\geq \text{Min} \left(\frac{(\overline{\text{Sum}/\text{Count}})^+ \times (\overline{\text{Sum}/\text{Count}})^+}{(\overline{\text{Sum}/\text{Count}})^- \times (\overline{\text{Sum}/\text{Count}})^-} \right). \end{aligned}$$

Count is always positive. By following Table 5 for computing sign bounds, we get $\overline{\left(\frac{\text{Sum}()}{\text{Count}()} \right)^2(\text{Cube}(X))}$:

8. There is another definition of Variance: $\text{Var}(X) = \frac{\sum_i (x_i - \bar{X})^2}{N-1}$. The discussions here also apply to this definition.

$$\text{Min} \left(\left(\frac{\text{Sum}^+(\text{Cube}(X))}{\text{Count}(\text{Cube}(X))} \right)^2, \left(\frac{\text{Sum}^-(\text{Cube}(X))}{\text{Count}(\text{Cube}(X))} \right)^2 \right).$$

In the above equation,

$$\overline{\text{Count}(\text{Cube}(X))} = \text{Sum}_i \text{Count}(X_i).$$

Moreover, based on Table 3,

$$\overline{\text{Sum}^+(\text{Cube}(X))} = \text{Min}_i \text{Sum}(X_i)$$

if $\text{Sum}(X_i) \geq 0$ for all $i = 1 \dots n$; otherwise, it is 0. Similarly, $\overline{\text{Sum}^-(\text{Cube}(X))} = \text{Max}_i \text{Sum}(X_i)$ if $\text{Sum}(X_i) < 0$ for all $i = 1 \dots n$; otherwise, it is 0.

In summary, we compute $\overline{\text{Var}(\text{Cube}(X))}$ as follows:

$$\begin{aligned} & \text{if } \text{Sum}(X_i) \geq 0 \text{ for } i = 1..n, \\ & \quad \frac{\text{Sum}_i \text{Sum}(X_i^2)}{\text{Sum}_i \text{Count}(X_i)} - \left(\frac{\text{Min}_i \text{Sum}(X_i)}{\text{Sum}_i \text{Count}(X_i)} \right)^2; \\ & \text{otherwise, if } \text{Sum}(X_i) < 0 \text{ for } i = 1..n, \\ & \quad \frac{\text{Sum}_i \text{Sum}(X_i^2)}{\text{Sum}_i \text{Count}(X_i)} - \left(\frac{\text{Max}_i \text{Sum}(X_i)}{\text{Sum}_i \text{Count}(X_i)} \right)^2; \\ & \text{otherwise,} \\ & \quad \frac{\text{Sum}_i \text{Sum}(X_i^2)}{\text{Sum}_i \text{Count}(X_i)}. \end{aligned}$$

With these equations, obviously tighter $\overline{\text{Var}(\text{Cube}(X))}$ is obtained if the sums of MSPs are of the same sign.

6 OPTIMIZATION FOR BOUNDING COMPLEX FUNCTIONS

When rewriting several commonly used algebraic aggregation functions, namely, Avg, Var, and Standard-Deviation, into arithmetic expressions of distributive aggregate functions, we notice that very often, they use the following subexpression:

$$F(X) = \text{Sum}_i G_1(X_i) / \text{Sum}_i G_2(X_i),$$

where G_1 and G_2 are some distributive aggregate functions, and $X_1, \dots, X_n$ are MSPs. (We can rewrite $\text{Avg}(X)$ as $\text{Sum}(X)/\text{Count}(X)$ and $\text{Var}(X)$ as

$$\text{QSum}(X)/\text{Count}(X) - (\text{Sum}(X)/\text{Count}(X))^2.$$

Moreover, we have

$$\text{Sum}(X)/\text{Count}(X) = \text{Sum}_i \text{Sum}(X_i) / \text{Sum}_i \text{Count}(X_i),$$

which is F , with $G_1 = \text{Sum}$ and $G_2 = \text{Count}$.) Therefore, it is desirable to give tighter bounds for F than those obtained by Table 4. The next result provides not only tight bounds but also the optimal.

Theorem 3. Let $\text{Cube}(X)$ be a data cube over measure X with MSPs $X_1, \dots, X_n$, let G_1 and G_2 be aggregate functions, and

let F be as defined above. If $G_2(X_i) > 0$ for all $i = 1 \dots n$, then we can bound F using

$$\overline{F(\text{Cube}(X))} = \text{Max}_i F(X_i), \underline{F(\text{Cube}(X))} = \text{Min}_i F(X_i).$$

Moreover, these bounds are the optimal bounds for $F(\text{Cube}(X))$.

Proof. Without loss of generality, we can assume that $\text{Min}_i F(X_i) = F(X_1)$, and $\text{Max}_i F(X_i) = F(X_n)$. Given a group g of $\text{Cube}(X)$, let $X_{g_1}, \dots, X_{g_k}$ be the MSPs contained in g . Thus, $g = X_{g_1} \cup \dots \cup X_{g_k}$. Then,

$$\begin{aligned} F(g) &= \text{Sum}_j G_1(X_{g_j}) / \text{Sum}_j G_2(X_{g_j}), \\ &= \frac{G_1(X_{g_1}) + \dots + G_1(X_{g_k})}{G_2(X_{g_1}) + \dots + G_2(X_{g_k})}, \\ &= \frac{F(X_{g_1}) \times G_2(X_{g_1}) + \dots + F(X_{g_k}) \times G_2(X_{g_k})}{G_2(X_{g_1}) + \dots + G_2(X_{g_k})}. \end{aligned}$$

The division in the last step is permitted, since $G_2(X_{g_i}) > 0$ for all i . Therefore,

$$\begin{aligned} & F(g) - F(X_n) \\ &= \frac{F(X_{g_1}) \times G_2(X_{g_1}) + \dots + F(X_{g_k}) \times G_2(X_{g_k})}{G_2(X_{g_1}) + \dots + G_2(X_{g_k})} \\ & \quad - F(X_n) \times \frac{G_2(X_{g_1}) + \dots + G_2(X_{g_k})}{G_2(X_{g_1}) + \dots + G_2(X_{g_k})} \\ &= \frac{(F(X_{g_1}) - F(X_n)) \times G_2(X_{g_1}) + \dots + (F(X_{g_k}) - F(X_n)) \times G_2(X_{g_k})}{G_2(X_{g_1}) + \dots + G_2(X_{g_k})}. \end{aligned}$$

Since $F(X_{g_j}) \leq F(X_n)$ and $G_2(X_{g_j}) > 0$ for all $j = 1 \dots k$, we have $G_2(X_{g_1}) + \dots + G_2(X_{g_k}) > 0$ and $(F(X_{g_j}) - F(X_n)) \times G_2(X_{g_j}) \leq 0$ for all j . Therefore, $F(g) - F(X_n) \leq 0$ or, equivalently, $F(g) \leq F(X_n)$. Similarly, it can be shown that $F(g) \geq F(X_1)$. Thus, we have proven that $F(X_n)$ and $F(X_1)$ are the upper and lower bounds for $F(\text{Cube}(X))$, respectively.

As $\text{Min}_i F(X_i)$ is the smallest aggregate for an MSP and thus is a real aggregate in $F(\text{Cube}(X))$, all other lower bounds for $F(\text{Cube}(X))$ must be greater than $\text{Min}_i F(X_i)$. $\text{Min}_i F(X_i)$ is thus the tightest lower bound for $F(\text{Cube}(X))$. Similarly, we can prove that $\text{Max}_i F(X_i)$ is the tightest upper bound for $F(\text{Cube}(X))$.

Next, we show that the bounds obtained by following Tables 2 and 4 are weaker. Considering $\text{Max}_i F(X_i)$ (and still assuming that $\text{Max}_i F(X_i) = F(X_n)$), the case for $\text{Min}_i F(X_i)$ is similar. By following Tables 2 and 4, $\overline{F(\text{Cube}(X))}$ is computed:

Case 1. There exists j such that $G_1(X_j) > 0$. $\overline{F(\text{Cube}(X))}$ is

$$\frac{\text{Sum}_i \overline{G_1(X_i)}}{\text{Sum}_i G_2(X_i)} = \frac{\text{Sum}_{G_1(X_i) \geq 0} G_1(X_i)}{\text{Min}_i G_2(X_i)}.$$

Since $G_2(X_i) > 0$ for all $i = 1 \dots n$, and $G_1(X_j) > 0$, $F(X_j) > 0$. Since $\text{Max}_i F(X_i) = F(X_n)$, it follows that $G_1(X_n) > 0$. Since $\text{Sum}_{G_1(X_i) \geq 0} G_1(X_i) \geq G_1(X_n)$ and $\text{Min}_i G_2(X_i) \leq G_2(X_n)$, we have

$$\frac{\text{Sum}_{G_1(X_i) \geq 0} G_1(X_i)}{\text{Min}_i G_2(X_i)} \geq \frac{G_1(X_n)}{G_2(X_n)} = F(X_n).$$

TABLE 6
Bounding Complex Aggregate Functions

$\overline{\text{Avg}(\text{Cube}(X))} = \text{Max}_i \text{Avg}(X_i), \text{Avg}(\text{Cube}(X)) = \text{Min}_i \text{Avg}(X_i)$
if there exist i and j such that $\text{Sum}(X_i)$ is positive and $\text{Sum}(X_j)$ is negative,
$\overline{\text{Var}(\text{Cube}(X))} = \text{Max}_i \frac{\text{Sum}(X_i^2)}{\text{Count}(X_i)},$
$\overline{\text{Var}(\text{Cube}(X))} = \text{Min}_i \frac{\text{Sum}(X_i^2)}{\text{Count}(X_i)} - \text{Max}_i ((\text{Max}_i \text{Avg}(X_i))^2, (\text{Min}_i \text{Avg}(X_i))^2);$
if $\text{Sum}(X_i) \geq 0$ for all i ($1 \leq i \leq n$)
$\overline{\text{Var}(\text{Cube}(X))} = \text{Max}_i \frac{\text{Sum}(X_i^2)}{\text{Count}(X_i)} - (\text{Min}_i \text{Avg}(X_i))^2,$
$\overline{\text{Var}(\text{Cube}(X))} = \text{Min}_i \frac{\text{Sum}(X_i^2)}{\text{Count}(X_i)} - (\text{Max}_i \text{Avg}(X_i))^2;$
if $\text{Sum}(X_i) \leq 0$ for all i ($1 \leq i \leq n$)
$\overline{\text{Var}(\text{Cube}(X))} = \text{Max}_i \frac{\text{Sum}(X_i^2)}{\text{Count}(X_i)} - (\text{Max}_i \text{Avg}(X_i))^2,$
$\overline{\text{Var}(\text{Cube}(X))} = \text{Min}_i \frac{\text{Sum}(X_i^2)}{\text{Count}(X_i)} - (\text{Min}_i \text{Avg}(X_i))^2.$

$\text{Cube}(X)$ is a data cube on measure X with MSPs $\{X_1, \dots, X_n\}$.

Hence, $\text{Max}_i F(X_i) = F(X_n)$ is a tighter bound.

Case 2. For all i ($1 \leq i \leq n$), $G_1(X_i) \leq 0$. $\overline{F(\text{Cube}(X))}$ is

$$\frac{\text{Sum}_i G_1(X_i)}{\text{Sum}_i G_2(X_i)} = \text{Max}_i G_1(X_i) / \text{Max}_i G_2(X_i).$$

As

$$0 \geq \text{Max}_i G_1(X_i) \geq G_1(X_n)$$

$$\text{and } 0 < G_2(X_n) \leq \text{Max}_i G_2(X_i),$$

$$0 \geq \text{Max}_i G_1(X_i) / \text{Max}_i G_2(X_i) \geq G_1(X_n) / G_2(X_n) = F(X_n).$$

Hence, $\text{Max}_i F(X_i) = F(X_n)$ is a tighter bound. $\square$

By following Theorem 3, tighter bounds for Avg and Var can be computed, as shown in Table 6.

In Section 7, we present an efficient implementation of the BP-Cubing algorithm, utilizing the bounding results discussed so far.

7 BOUND-PRUNE CUBING ALGORITHMS

We first present the group tree (G-tree) data structure that we will use for iceberg cubing. We then explain how bound prune cubing (BP-Cubing) is implemented on the G-tree. Finally, we briefly discuss how our algorithms utilize antimonotone constraints. The iceberg cube with the constraint “Avg(Sale) in [15, 20]” on the Sales data set in Table 1 is used as a running example throughout this section.

7.1 The G-Tree

The underlying data structure for BP-Cubing is the G-tree. A G-tree is the compression of a given input data set and is constructed by one scan of the data set. It is used for both the top-down and bottom-up bound prune cubing algorithms (BP-Cubing(TD) and BP-Cubing(BU), respectively), although there are some minor differences depending on the traversal strategy.

The G-tree for the data set in Table 1 is shown in Fig. 3. A G-tree for an n -dimensional data set is of depth n , where each level represents a dimension. A path starting from the root collapses the tuples with common dimension values along the path. Each tree node keeps

Fig. 3. A G-tree for the data set in Table 1.

the auxiliary aggregates necessary to compute the iceberg cube. In our example, the auxiliary aggregates in a node are $\text{Sum}(\text{Sale})$ and $\text{Count}(\ast)$. For the leftmost path from the root of the G-tree in Fig. 3, the node (March) shows that there are 70 tuples with $\text{Sum}(\text{Sale}) = 300$ in the (March, *, *, *) partition, whereas the node (Peter) shows that there are 40 tuples with $\text{Sum}(\text{Sale}) = 100$ in the (March, TV, Peter, *) partition.

For a given data set, different dimension orders result in different G-trees. Depending on whether the G-tree is traversed top-down or bottom-up in computing cubes, the tree should be constructed in different orders of dimensions. Antimonotone pruning is effective when the most discriminating dimension is examined first. This suggests the cardinality-descending order of dimensions during cube computation. As a result, for bottom-up traversal, the cardinality-ascending order should be used in constructing the G-tree. In contrast, for top-down traversal, the cardinality-descending order should be used in constructing the G-tree. Our experiments have confirmed that such heuristics indeed have a positive impact on the effectiveness of pruning and the efficiency of cubing algorithms.

7.2 Top-Down Bound-Prune Cubing on G-Trees

An observation of Fig. 3 reveals some useful facts: The path from the root to a node of the G-tree aggregates a group, and each level of the G-tree computes a group-by of the cube lattice. For the G-tree in Fig. 3, the root node has the aggregate for the group (*, *, *, *), with $\text{Count}(\ast) = 88$, and $\text{Sum}(\text{Sale}) = 700$. The nodes at level 1 compute the aggregates for groups in (Month, *, *, *), which are (March, *, *, *, 70, 300), (January, *, *, *, 5, 200), and (April, *, *, *, 13, 200). The nodes at the next three levels compute the aggregates for the group-bys (Month, Product), (Month, Product, SalesMan), and (Month, Product, SalesMan, City), respectively. The leaf nodes give the MSPs for $\text{Cube}(\text{Month}, \text{Product}, \text{SalesMan}, \text{City})$. We summarize these in the following observations.

Observation 2. In a G-tree, the aggregates in each node are the aggregates for the group with dimension values on the path from the root to the node.

Observation 3. In a G-tree, for n dimensions $A_1, \dots, A_n$, the leaf nodes are the MSPs for $\text{Cube}(A_1, \dots, A_n)$.

Fig. 4. Top-down bound prune cubing of $\text{Cube}(ABCD)$: For each node, we show a G-tree (before “:”), the group-bys (after “:”) computed in the G-tree, and the shared dimensions (after the “/”). The numbers show the order in which trees are built.

G-trees are constructed to compute all group-bys in a data cube. When constructing a G-tree for an n -dimensional data set, we simultaneously compute n group-bys, namely, the group-bys whose dimensions are prefixes of the list of dimensions ordered by the levels of the tree. To compute the other group-bys in the cube, we collapse one dimension from a given G-tree at a time to construct a sub-G-tree⁹ and compute the corresponding group-bys of the sub-G-tree. For example, by collapsing dimension *Product* in the original G-tree, given in Fig. 3, we get the subtree $\{\text{Month}, (-\text{Product}), \text{SalesMan}, \text{City}, \text{Customer}\}$, shown in Fig. 5.

Fig. 4 shows the (sub)G-trees constructed for computing the data cube on dimensions *A*, *B*, *C*, and *D*. Each node in Fig. 4 represents a G-tree and the set of all corresponding group-bys. The ABCD tree at the top is constructed by one scan of the data set, and the corresponding group-bys (*A*, *B*, *C*, *D*), which are the (*A*, *B*, *C*), (*A*, *B*), (*A*), and () trees are computed during this scan. The sub-G-trees of the ABCD tree, which are the $(-A)BCD$, $A(-B)CD$, and $AB(-C)D$ trees, are formed by collapsing dimensions *A*, *B*, and *C*, respectively. The CD tree is a subtree of the BCD tree and recursively is also a subtree of the ABCD tree. The dimensions after “/” in the nodes denote common prefix dimensions for the tree at the node and all of its subtrees. *A* is the common prefix dimension for the ACD tree and its subtrees. All group-bys that are computed on the ACD tree and its subtrees form subcubes for *A* values $\text{Cube}(CD)|a_i$. The leaf nodes originating from a_i are the MSPs for $\text{Cube}(CD)|a_i$. In the sub-G-tree construction process, we also compute the bounds and use them for pruning, as described in the observation below.

Observation 4. With top-down aggregation, given a G-tree G with n dimensions $A_1, \dots, A_n$ and a subtree G_k , by collapsing a dimension A_k , $1 < k < n$, $A_1, \dots, A_{k-1}$ are the common prefix dimensions for G_k and all its subtrees. Each node $(a_1, \dots, a_{k-1})$ of G gives the prefix dimension values for $\text{Cube}(A_{k+1}, \dots, A_n)|a_1, \dots, a_{k-1}$. The core of the cube consists of the MSPs corresponding to the leaf nodes of the branches originating from the node $(a_1, \dots, a_{k-1})$. If the bounds of $\text{Cube}(A_{k+1}, \dots, A_n)|a_1, \dots, a_{k-1}$ fail the given constraint, then those branches can be pruned.

Example 9. Consider the G-tree G in Fig. 3 as the original tree and the G-tree G_p in Fig. 5 as the subtree. *Month* is the prefix dimension for the group-bys on G_p and the subtree of G_p . The subcubes are

9. It should be pointed out that a sub-G-tree is not part of an original G-tree but is obtained by collapsing a given dimension of the original G-tree.

Fig. 5. The subtree on $\{\text{Month}, (-\text{Product}), \text{SalesMan}, \text{City}\}$ obtained by collapsing dimension *Product* in the original G-tree, given in Fig. 3.

1. $\text{Cube}(\text{SalesMan}, \text{City})|\text{March}$,
2. $\text{Cube}(\text{SalesMan}, \text{City})|\text{January}$, and
3. $\text{Cube}(\text{SalesMan}, \text{City})|\text{April}$.

In G , the leaf nodes of the March subtree are the MSPs of $\text{Cube}(\text{SalesMan}, \text{City})|\text{March}$. With the constraint “ $\text{Avg}(\text{Sale})$ in $[15, 20]$,” G is pruned in constructing the subtree G_p . By following Table 6, the bounds for $\text{Cube}(\text{SalesMan}, \text{City})|\text{March}$ are computed from the three leaf nodes originating from node (March) of G :

$$\begin{aligned} \text{Avg}(\text{Cube}(\text{Sale})|\text{Mar}) &= \text{Max}(\{100/40, 100/20, 100/10\}) \\ &= 10. \end{aligned}$$

$$\begin{aligned} \text{Avg}(\text{Cube}(\text{Sale})|\text{Mar}) &= \text{Min}(\{100/40, 100/20, 100/10\}) \\ &= 2.5. \end{aligned}$$

As $[2.5, 10]$ violates “ $\text{Avg}(\text{Sale})$ in $[15, 20]$,” all three branches originating from (March) are removed from further computation. Similarly, $\text{Avg}(\text{Sale})$ of $\text{Cube}(\text{Salesman}, \text{City})|\text{January}$ is bounded as $[40, 40]$, which violates the constraint, and is pruned. $\text{Avg}(\text{Sale})$ of $\text{Cube}(\text{Salesman}, \text{City})|\text{April}$ is bounded as $[12.5, 25]$, which does not violate the constraint, and the branches from (April) are kept in G_p . The nodes in dashed lines in Fig. 5 highlight the fact that the branches from (March) and (January) are pruned before the G_p tree is generated, and importantly, they are permanently pruned from all future recursive computation.

7.3 The Top-Down Bound-Prune Cubing Algorithm

In this section, we present the Top-Down Bound-Prune Cubing (BP-Cubing(TD)) algorithm, shown in Algorithm 1, for computing iceberg data cubes. It performs bound-pruning in the top-down multiway aggregation manner, and it uses the G-trees.

Algorithm 1. The Top-Down Bound-Prune Cubing Algorithm

Input: A data set D over dimensions $A_1, \dots, A_n$ and aggregation constraint C , assumed global.

Output: An n -dimensional iceberg cube on D satisfying C .

- (1) Build the G-tree $T(A_1, \dots, A_n)$ from D ;
- (2) Output aggregates satisfying C , computed when T was built;
- (3) **for** $i = 1 \dots (n - 1)$, **do**
- (4) **BP-Cubing** $(T(A_1, \dots, A_n), A_i)$;

Fig. 6. The G-tree for the data set in Table 1, with header table and side links.

// B_i , the i th dimension of T is the dimension to be collapsed
Procedure BP-Cubing ($T(B_1, \dots, B_k), B_i$)

- (5) $T_s \leftarrow \text{nil}$;
 //construct subtree T_s by collapsing B_i .
- (6) **for each** node n_{i-1} of dimension B_{i-1} in T , **do**
- (7) $V_{i-1} \leftarrow$ dimension values on the path from root to n_{i-1} ;
 // the MSPs for $\text{Cube}(B_{i+1}, \dots, B_k) | V_{i-1}$
- (8) Let M be the set of leaves originating from n_{i-1} ;
- (9) Compute the bounds for $\text{Cube}(B_{i+1}, \dots, B_k) | V_{i-1}$ from M ;
 // prune the paths if the bounds violate C
- (10) **if** the bounds do not violate C **then**
- (11) Collapse B_i and add the paths passing n_{i-1} of T to T_s ;
- (12) Output aggregates on T_s that satisfy C ;
- (13) **for** $j = (i + 1) \dots (k - 1)$ **do**
- (14) **BP-Cubing**($T_s(B_1, \dots, B_{i-1}, B_{i+1}, \dots, B_k), B_j$)

Let D be a data set with n dimensions $A_1, \dots, A_n$ and C be the iceberg constraint. To compute the iceberg cube, first, a G-tree T is built by one scan of D (line 1). As mentioned in Observation 2, we simultaneously aggregate n group-bys, namely, (A_1) , $(A_1, A_2), \dots, (A_1, A_2, \dots, A_n)$, when building the G-tree. Those groups that satisfy C are output (line 2).

The Bound-Prune Cubing (BP-Cubing) procedure computes the iceberg cube by recursively collapsing dimensions and building sub-G-trees. Given a G-tree, before a subtree is built, bounds for branches are computed, and those branches whose bounds fail the constraint are pruned from further computation (lines 9-11). The leaf nodes of a G-tree are the MSPs for the G-tree, and these MSPs are used to compute the corresponding aggregate bounds. T_s is a subtree of T , obtained by collapsing dimension B_i ; the collapsing computation is described in lines 6-11. Aggregates are computed during the construction of T_s . Then, BP-Cubing is recursively called (lines 13-14), where group-bys of dimensions after B_i are computed.

7.4 Bottom-Up Bound-Prune Cubing on G-Trees

The BP-Cubing(BU) algorithm is similar to Algorithm 1. With the bottom-up cubing strategy, a group is computed before its subgroups. There are several differences concerning 1) the G-tree used, 2) how sub-G-trees are constructed, and 3) how bounding with MSPs are done. We describe these below.

Fig. 7. The subtree of the G-tree in Fig. 6, with City = Sydney.

We first consider the G-tree used. In BUC, a G-tree will also contain a header table and side links. A header table entry represents a one-dimensional group together with the corresponding aggregates, and it is linked to nodes with the corresponding dimension value. Fig. 6 shows such a G-tree for the sample Sales data set. The side-links will be used to efficiently derive aggregates of other groups in subsequent computation.

We now consider how sub-G-trees are constructed. In the first G-tree for a data set, the header table contains one-dimensional groups. In the sub-G-trees constructed from a given G-tree, the header table contains groups whose dimensionality is one level higher than that of groups for the given G-tree. For the G-tree in Fig. 6, we build sub-G-trees for the 2D subgroups of the (City) groups, namely, the groups of (Salesman, City), (Product, City), and (Month, City). Similarly, we build sub-G-trees for the 3D subgroups of the (Salesman, City) groups, and so on. The sub-G-trees are obtained by merging paths in the original G-tree. For example, to compute the 2D subgroups of Sydney, the subtree in Fig. 7 is constructed by merging the paths from the root to the nodes on the Sydney side link. In this process, the aggregates for the nodes above the (Sydney) nodes need to be modified to remove the contribution of the non-Sydney cities. For example, the node (John) has the aggregates of (30, 200) for both Perth and Sydney but only has the aggregates of (10, 100) for Sydney.

We now turn to bounding with MSPs. Recall that, with the top-down aggregation strategy, all MSPs for a (sub) data cube are “conveniently available” for bounding (by “conveniently available,” we mean that they are available without extra overhead). In the bottom-up aggregation, the “conveniently available” MSPs are those pointed to by side links of certain header table entries. The observation below describes how bounds are computed, and pruning is achieved.

Observation 5. For a G-tree of dimensions $A_1, \dots, A_n$, the leaf nodes are the MSPs for $\text{Cube}(A_1, \dots, A_n)$. With the BUC strategy, for $a_k \in \text{domain}(A_k) (1 \leq k \leq n)$, the subgroups of (a_k) to be computed from the branches on the side link of a_k comprise $\text{Cube}(A_1, \dots, A_{k-1}) | a_k$, and MSPs of the cube are the leaf nodes on the side link for a_k (available for bounding). The subtrees from the side link of a_k can be pruned if the bounds of $\text{Cube}(A_1, \dots, A_{k-1}) | a_k$ fail the given constraint.

Example 10. With $\text{Cube}(\text{Month}, \text{Product}, \text{Salesman}) | \text{Sydney}$, shown in Fig. 6, its MSPs are the leaf nodes (Sydney, 10, 100) and (Sydney, 5, 100) on the side link for Sydney. Thus,

Fig. 8. Bottom-up bound prune cubing (BP-Cubing) on four dimensions A, B, C, and D

$$\text{Cube}(\text{Month}, \text{Product}, \text{Salesman})|_{\text{Sydney}} = 100/5 = 20,$$

$$\text{Cube}(\text{Month}, \text{Product}, \text{Salesman})|_{\text{Sydney}} = 100/10 = 10.$$

As $[10, 20]$ does not violate the constraint “Avg (Sale) in $[15, 20]$,” the subtree for Sydney (shown in Fig. 7) is thus created.

In Fig. 8, we use an example to give some high-level view of the BUC process for a four-dimensional data cube. Each node represents the group that is computed on some G-tree. From top to bottom, nodes are linked by the subtree relationship. The superscripts over the nodes indicate the order in which the G-trees are created. With a tree, the dimensions after “/” are the shared dimensions for the tree and all its subtrees. They are also the conditional dimensions where the tree is created. Generally, the BP-Cubing(BU) algorithm is given in Algorithm 2.

Algorithm 2. The Bottom-Up Bound-Prune Cubing Algorithm

Input: A data set D over dimensions $A_1, \dots, A_n$ and aggregation constraint C .

Output: The n -dimensional iceberg cube on D , satisfying C .

- (1) Build the G-tree $T(A_1, \dots, A_n)$ from D ;
- (2) Output all aggregates in $T(A_1, \dots, A_n).Header$;
- (3) **foreach** $a \in T.Header$, **do**
- (4) **BP-Cubing**($T(A_1, \dots, A_n), \{a\}$);

Procedure BP-Cubing ($T(B_1, \dots, B_k), S$)

// S is a set of dimension values as the condition for subcubes.

- (5) $T_s = \text{nil}$;
- (6) Suppose a is on the i^{th} dimension of T ;
- (7) Let M be the nodes following the side link of a ;
- (8) Compute the bounds for $\text{Cube}(B_1, \dots, B_{i-1})|_S$ from the leaves originating from M ;
- (9) **if** the bounds do not violate C , **then** // pruning
 // Section 7.4
- (10) Construct the subtree $T_s(B_1, \dots, B_{i-1})$;
- (11) Output aggregates on $T_s.Header$ that satisfy C ;
- (12) **foreach** $a_s \in T_s.Header$ **do**
- (13) **BP-Cubing**($T_s(B_1, \dots, B_{i-1}), S \cup \{a_s\}$);

7.5 Interaction with Antimonotone Constraints

Our discussions have focused on complex nonantimonotone aggregation constraints. Antimonotone constraints can be easily incorporated in BP-Cubing as follows: Suppose an antimonotone constraint C' is present, in addition to the nonantimonotone constraint C . First, in lines 2 and 12 of Algorithm 1, a group g is checked against both C and C'

TABLE 7
Census, TPC-R, and Weather Multidimensional Data Sets

Dataset	Cardinality(Range: Average)	# Dimensions	# records
Census	[7..48]: 14	12	88,443
TPC-R	[3..25]: 10	8	10^6
Weather	[2..6505]: 812	9	10^5

before being output. More importantly, before the bounds are computed at line 9, the branches of T originating from n_i are pruned from further computation if n_i fails C' , as all subgroups of n_i will also fail C' . With top-down cubing, the prefix groups before the collapsing dimension that fail C' are pruned. With bottom-up cubing, header-table entries that fail C' are pruned from further computation.

8 EXPERIMENTS

In this section, we evaluate the performance of both the BP-Cubing(TD) and BP-Cubing(BU) algorithms with experiments. We compare the performance of these algorithms with that of the DnA algorithm [14], [15] on non-antimonotone aggregation constraints and also with that of the BUC algorithm [4] on antimonotone constraints.

DnA is recent work on pruning for nonantimonotone aggregation constraints. As DnA is not designed for pruning for antimonotone constraints, extra processing is needed to prune for such constraints. On the other hand, BUC uses the same partition-based bottom-up aggregation strategy as DnA and is designed for antimonotone constraints. Thus, BP-Cubing is compared with BUC on pruning with antimonotone constraints.

To do fair comparison, all algorithms were implemented with all possible optimization techniques. BUC was implemented with the collapsing duplicates optimization [4]. We added collapsing duplicates and indexing tuples to DnA, even though such optimizations were not reported in the original papers [14], [15]. Block memory allocation is used in the BP-cubing algorithms to reduce the number of calls of the dynamic memory allocation functions. It is assumed that, for all algorithms, data structures used can fit into memory. All experiments were performed on a PC with an i686 processor running GNU/Linux. As outputting the groups can take a significant amount of time for big data cubes, we choose to exclude the time for output in the timing for the algorithms. The thresholds for constraints are selected in a way such that there are at least 10 groups in the output. We experimented with constraints involving the aggregate functions Count, Avg, Var, and Sum.

8.1 Data Sets

We used real-world, as well as artificial, data sets in our experiments. When selecting the data sets, we considered the following data characteristics: dense versus sparse and random versus skewed. Table 7 summarizes the data sets used in our experiments.

The US census data set¹⁰ was collected in a 1990 US households survey. The original data set had 61 attributes such as `hrswork1` (hours worked last week), `nchild` (number of own children on the household), and `valueh` (value of house). We selected 12 discrete attributes as

10. <http://ftp.ipums.org/ipums/data/ip19001.Z>.

Fig. 9. BP-Cubing versus BUC on the “Count(*) ≥ α ” constraint. (a) Census. (b) TPC-R. (c) Weather. (d) Weather-5.

dimensions and a numerical attribute as the measure. The data set is dense and skewed.

The Weather data set¹¹ contains real weather reports from various weather stations in 1985. We used these nine attributes as dimensions: station-id, longitude, solar-altitude, latitude, present-weather, weather-change-code, day, hour, and brightness. Their cardinalities, respectively, are 6,505, 351, 179, 152, 99, 10, 8, 3, and 2. We randomly generated values between 1 and 100 as the measure. This Weather data set is the same as that used in the experiments in [4], where BUC was shown to be efficient for sparse data.

The TPC-R data set¹² is an artificial data set provided by the Transaction Processing Council and is designed for testing the performance of representative complex queries in high-level business decision-making environment. Its dimensions include customer, supplier, order, and shipment. The original TPC-R data set consists of several relational tables. We constructed a joined relation as our multidimensional data set. A numerical attribute is selected as the measure. The TPC-R data set is relatively dense and random.

8.2 BP-Cubing versus BUC on “Count(*) ≥ α ”

This set of experiments is designed to evaluate two aspects of BP-Cubing: the tree-based aggregation strategy and pruning with simple antimonotone constraints. This involves comparing the performance of the BP-Cubing algorithms with BUC on computing iceberg cubes for the constraint “Count(*) ≥ α .” For the special case of $\alpha = 1$, the full data cubes are computed. Since there is no pruning, the difference in efficiency should be solely attributed to the aggregation strategies. Fig. 9 shows the runtime of the algorithms.

Fig. 9a shows that BUC uses 69.31 seconds to compute the full data cube over the dense and skewed Census data set, and BP-Cubing(BU) and BP-Cubing(TD) take 35.67 and 18.39 seconds, respectively, which are 1.9 and 3.76 times

faster than BUC. Fig. 9b shows that BUC uses 87.97 seconds to compute the full cube for the TPC-R data set. In contrast, BP-Cubing(BU) and BP-Cubing(TD) use only 17.9 and 5.89 seconds, respectively, which are 4.9 and 14.9 times faster. On the other hand, BUC is more efficient on the sparse Weather data than the BP-Cubing algorithms. Fig. 9c shows that BUC uses 6.16 seconds on computing the full cube, whereas BP-Cubing(BU) and BP-Cubing(TD) use 21.82 and 19.48 seconds, respectively. When executed on the data set obtained by removing the four dimensions with the largest cardinalities of 6,505, 351, 179, and 152 from the Weather data set, the BP-Cubing algorithms and BUC are all able to compute the full cube quickly: BUC uses 0.3 seconds, whereas both BP-Cubing algorithms use 0.02 seconds, as shown in Fig. 9d.

This set of experiments has shown that top-down multiway aggregation using G-trees is a superb aggregation strategy, and the experimental results suggest that tree-based aggregation is generally more efficient than recursive partition-based aggregation. Our experiments also confirmed the observation in [4] that partition-based bottom-up aggregation is suitable for computing sparse data cubes.

Even though BP-Cubing(BU) and BUC both use the same bottom-up strategy, BP-Cubing(BU) has better performance than BUC when dealing with antimonotone constraints. This implies that aggregation on G-trees is a superior strategy. In Fig. 9c, we see that initially, BP-Cubing(TD) is more efficient than BP-Cubing(BU) for computing the full cube on Weather. However, for constraints with the threshold $\alpha \geq 5$, BP-Cubing(BU) outperforms BP-Cubing(TD). This may have happened because the size of the G-tree is large for sparse data, and the top-down aggregation strategy may have aggregated many groups, which turns out to fail the constraint.

8.3 BP-Cubing versus DnA on “Avg(X) in $[\delta_1, \delta_2]$ ”

In this set of experiments, we compare the performance of the BP-Cubing algorithms against DnA on the nonantimonotone constraint “Avg(X) in $[\delta_1, \delta_2]$.”

Fig. 10 shows that the BP-Cubing algorithms scale very well when the $[\delta_1, \delta_2]$ range threshold becomes looser. In all data sets, the BP-Cubing algorithms show modest linear increase in computation time. In contrast, the performance of DnA degrades significantly (which may be due to the fact that its pruning is at the tuple record level: When many groups need to be processed, the search cost for the minimal partition to approximate a group becomes high (see Section 9 for more discussions)).

Figs. 10a and 10b show that in the dense data sets Census and TPC-R, the BP-Cubing algorithms significantly outperform DnA, and the BP-Cubing algorithm shows the best performance. For the constraint “Avg(X) in [500, 50,000]” over Census, DnA finishes in 65.04 seconds, whereas BP-Cubing(TD) finishes in only 3.53 seconds, which is 18.42 times faster. At the lower end, for the constraint “Avg(X) in [500, 10,000],” BP-Cubing(TD) is 7.09 times faster. In TPC-R, BP-Cubing(TD) outperforms DnA by 4.62–6.22 times. BP-Cubing(BU) also outperforms DnA overall, especially for larger constraint ranges. For the constraint “Avg(X) in [500, 50,000]” over Census, BP-Cubing(BU) is 2.33 times faster than DnA.

Fig. 10c shows that in the sparse Weather data set, BP-Cubing(BU) achieves more significant efficiency improvement over DnA than BP-Cubing(TD). In this data set,

11. <http://cdiac.ornl.gov/ftp/ndp026b/SEP85L.DAT.Z>.

12. <http://www.tpc.org/tpcr/>.

Fig. 10. BP-Cubing versus DnA on the “Avg(X) in $[\delta_1, \delta_2]$ ” constraint. (a) Census. (b) TPC-R. (c) Weather.

the figure may suggest that the improvement of the BP-Cubing algorithms over DnA is not as pronounced as in Census and TPC-R. However, Fig. 9 has shown that in the sparse Weather data, the partition-based aggregation strategy of DnA is more efficient than the tree-based aggregation strategy of BP-Cubing. With an aggregation strategy that works not that well, the BP-Cubing algorithms still achieve better performance than DnA. Such dramatic result can only be attributed to the effectiveness of bound pruning.

As experiments have shown that BP-Cubing(TD) has better performance than BP-Cubing(BU) in general, BP-Cubing(TD) is used in later experiments for comparing with DnA.

8.4 BP-Cubing versus DnA on “Var(X) $\geq \alpha$ ”

In this section, we compare our bounding techniques against DnA on the constraint “Var(X) $\geq \alpha$,” which involves the complex nonmonotone function Var. For BP-Cubing, we obtain the upper bound for Var by following Table 6. For DnA, we derive the upper bound for Var as follows ([14, Example 4.2]):

$$\frac{QSUM(X)}{\text{Count } 1(c)} - \left(\frac{\text{psum } 1(c)}{\text{Count } 2(X)} \right)^2 - \left(\frac{\text{nsum } 1(c)}{\text{Count } 3(X)} \right)^2 + 2 \times \frac{\text{psum } 2(X) \times \text{nsum } 2(X)}{(\text{Count } 4(c))^2},$$

whose notations are explained later in Section 9.

The runtime of both BP-Cubing(TD) and DnA is shown in Figs. 11a, 11b, and 11c. When the Var threshold gets larger, the iceberg cubes get smaller, and both algorithms finish faster. BP-Cubing is always faster than DnA at all Var thresholds. The speedup is usually around several times. This can be attributed to the tighter bounds obtained using MSPs.

Fig. 11. BP-Cubing versus DnA on “Var(X) $\geq \alpha$ ” and “Sum(X) $\geq \beta$.” (a) Var(X) $\geq \alpha$: Census. (b) Var(X) $\geq \alpha$: TPC-R. (c) Var(X) $\geq \alpha$: Weather. (d) Sum(X) $\geq \beta$: Census. (e) Sum(X) $\geq \beta$: TPC-R. (f) Sum(X) $\geq \beta$: Weather.

8.5 BP-Cubing versus DnA on “Sum(X) $\geq \beta$ ”

We now compare BP-Cubing with DnA on computing iceberg cubes defined by “Sum(X) $\geq \beta$.” As shown in Table 2, Sum(X) is a representative for our general bounding theory, which does not involve any optimization. The upper bound of Sum(X) needs to be computed for pruning. In BP-Cubing, this is computed following Table 2. In DnA, this is computed following [15] (see Section 9). The artificial data sets TPC-R and Weather contain only positive measure values, which renders Sum(X) $\geq \beta$ an antimonotone constraint. We randomly negated 10 percent of the measure values to make the constraint nonantimonotone so that pruning techniques can be sensibly compared. Figs. 11d, 11e, and 11f demonstrate the runtime of both algorithms on the three data sets under different values of β . BP-Cubing(TD) significantly outperforms DnA at all thresholds for Sum(X).

To ascertain the contribution of pruning toward the efficiency gain of BP-Cubing over DnA, for each algorithm, we collected the number of groups that are examined for pruning, namely, groups whose aggregates are bounded (approximated) and tested against the given constraint. There are two types of examined groups: a “true positive” (or nonsolution group) is a group whose approximated aggregate and its real aggregate pass the constraint, and a

TABLE 8
Number of Examined Groups versus Number of Solution Groups on $\text{Sum}(X) \geq \beta$

Census ($\beta = 10^3 \dots 10^8$)			TPC-R ($\beta = 10^4 \dots 10^9$)			Weather ($\beta = 400 \dots 900$)		
Solution	BPC*	DnA	Solution	BPC*	DnA	Solution	BPC*	DnA
19,619,534	20,405,004	21,378,725	4,068,317	4,265,176	4,316,109	406,628	1,830,753	8,027,054
14,782,195	16,324,797	18,609,867	2,962,327	3,872,446	4,293,431	297,216	1,526,206	7,379,501
3,736,088	5,662,047	10,087,280	775,860	1,829,161	3,690,935	232,948	1,274,132	6,695,759
644,205	1,432,753	4,994,413	102,951	244,951	1,707,564	185,441	1,059,187	6,041,456
72,459	233,714	2,195,618	8,593	32,227	1,158,332	148,660	852,296	5,271,776
4,137	20,246	658,320	357	3,383	1,014,269	119,401	659,192	4,438,490

*Bound-prune cubing denotes (BP-Cubing(TD)).

“false positive” (or solution group) is a group whose approximated aggregate passes the constraint, but whose real aggregate fails the constraint.

Table 8 shows the number of examined groups versus the number of solution groups for the three data sets. The three columns list, respectively, the number of solution groups in the output and the number of groups examined in BP-Cubing(TD) and DnA. Substantially fewer groups are examined in BP-Cubing(TD) at all thresholds. This shows that BP-Cubing with MSPs has a clear advantage. Observe that, when the number of solution groups decreases, the numbers of groups examined by both algorithms decrease as well, but that number in DnA decreases at a much slower rate than that in BP-Cubing(TD).

9 RELATED WORK

DnA [14], [15] is a recent approach for pruning with nonantimonotone aggregation constraints in iceberg cubing. The main idea of DnA is to divide a partition of tuples into two subspaces of positive and negative measure values, respectively, so that a given constraint can be rewritten using antimonotone or monotone constraints in subspaces. For example, consider a measure X , which can be positive or negative. Given the constraint $\text{Avg}(X) \geq \alpha$, the authors would first rewrite the original aggregate into

$$\text{Avg}(X) = \text{psum}(X)/\text{Count}_1(X) - \text{nsum}(X)/\text{Count}_2(X),$$

where psum and nsum are the (absolute) sum of positive/negative X values, and Count_1 and Count_2 are rewriting Count in the positive and negative spaces. Then, they would use $\text{psum}(X)/\text{Count}_1(c) - \text{nsum}(c)/\text{Count}_2(X)$ as a weaker antimonotone approximator for $\text{Avg}(X)$, where c is some smallest subpartition of the given partition. If the approximate fails the threshold α , then all groups that are subgroups of X and supergroups of c are pruned.

BP-Cubing differs from DnA in several aspects:

1. Rather than the separately monotone rewriting strategy of DnA, in BP-Cubing, rewriting follows the principles that an algebraic aggregate function can be expressed as an algebraic expression of distributive aggregate functions and that the aggregate value for a group can be computed from the aggregates of MSPs. For example, given measure X with MSPs $X_1, \dots, X_n$, $\text{Avg}(X)$ is rewritten into

$$\begin{aligned} \text{Sum}(X)/\text{Count}(X) = \\ \text{Sum}_i(\text{Sum}(X_i))/\text{Sum}_i(\text{count}(X_i)). \end{aligned}$$

2. In BP-Cubing, the optimization for complex aggregate functions such as Avg and Var can produce very tight bounds. Continuing with the previous example, by following Table 6, the optimized upper bound for $\text{Avg}(X)$ is $\text{Max}(\{\text{Avg}(c_i)\})$, where c_i iterates over the smallest partitions. As $\text{Avg}(c_i)$ is the real aggregate of a group in the search space, $\text{Max}(\{\text{Avg}(c_i)\})$ is the optimal approximator and is tighter (smaller) than

$$\text{psum}(X)/\text{Count}_1(c) - \text{Sum}(c)/\text{Count}_2(X),$$

which is the bound derived by DnA.

3. MSPs are nonempty groups of tuples and BP-Cubing prunes groups. Moreover, the G-tree structure of BP-Cubing greatly facilitates top-down multiway aggregation and saves computation, especially for dense data. DnA calculates aggregate bounds from tuples and prunes tuples—the bottom-up recursive partitioning aggregation strategy can incur extra cost searching for and pruning tuples that do not occur in any groups that satisfy a given constraint.
4. Extra processing is needed in DnA to incorporate antimonotone constraints such as “ $\text{Count}(*) \geq n$.” Continuing with the previous example, consider the constraint “ $\text{Avg}(X) \geq \alpha$ and $\text{Count}(*) \geq n$ ” and the (ab) partition with c, d , and e as further partitioning dimension values. In DnA, with recursive partitioning, even if (e) is infrequent, subpartitions of (ab) and (e), namely, (abe), (abce), (abde), and (abcde), are not pruned (the Rollback tree was proposed to prune these groups). Note that these are only some of the subpartitions of (e) that should be pruned. In BP-Cubing, as discussed in Section 7.5, all subpartitions of a partition failing an antimonotone constraint are pruned.

The top- k average technique [7] was designed specifically for the constraint “ $\text{Avg}(x) \geq \alpha$ and $\text{Count}(*) \geq k$ ” and is not a general pruning technique for iceberg cubing. The tree-based bottom-up aggregation strategy was first proposed in [7]. Top-down aggregation was proposed independently in [16] for performing multiway aggregation in cubing; however, the work was focused on how to incorporate bottom-up pruning for antimonotone aggregation constraints into top-down aggregation. The BUC algorithm was proposed in [4], where pruning with antimonotone constraints was performed in the bottom-up recursive partitioning aggregation framework.

Our study is also related to constraint data mining [2], [3], [8], [9], [12], [13]. Agrawal and Srikant [2] wrote the first paper on pruning with the antimonotone constraint $\text{Count}(*) \geq \alpha$ for association mining. In other works on

association mining, pruning strategies with specific constraints were proposed, namely, minimum improvement constraints [3], succinct constraints [8], convertible constraints [9], item constraints [12], and support constraints [13]. These constraints are orthogonal to the aggregation constraints that we consider in iceberg cubing.

10 CONCLUSIONS

In computing iceberg cubes, pruning with nonantimonotone aggregation constraints has been a challenging problem. In this paper, we have proposed pruning with nonantimonotone constraints by estimating their upper and lower bounds on a data cube from aggregates of the most specific (or minimal) partitions. We have proposed iceberg cubing algorithms, called BP-Cubing, which incorporate bounding and pruning, and incorporated top-down and bottom-up aggregation strategies by using the G-tree structure. Our extensive experiments on real and artificial data sets have shown that BP-Cubing is effective for pruning under various data characteristics, and our iceberg cubing algorithms significantly outperform state-of-the-art iceberg cubing algorithms.

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