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Perturbation-based Imperfection Analysis for Composite Cylindrical Shells Buckling in Compression

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Abstract

A numerical investigation was conducted into a perturbation-based analysis approach for assessing the imperfection sensitivity of composite cylindrical shells buckling under compression loading. The Single Perturbation Load Analysis (SPLA) approach was applied, which uses a single lateral load to introduce a realistic, worst-case and stimulating imperfection pattern. Finite element analysis was conducted for cylinders of both monolithic composite laminate and sandwich construction, with and without small and large cutouts. It was found that using a perturbation displacement equal to the shell thickness provides a suitable technique for estimating the reduction in buckling load caused by imperfections. Predictions of buckling knockdown factors using the SPLA approach showed advantages over the use of eigenmodes as the SPLA approach provides a clear design point and does not require experimental data for calibration. The effect of small and large cutouts was analogous to the effect of small and large perturbation loads. The location of the perturbation load influenced the buckling knockdown factors for both small and large cutouts, and worst-case locations were identified for both configurations.

Keywords: Shell buckling, imperfection sensitivity, finite element analysis, knockdown factor

1. Introduction

Prediction of the buckling loads of thin-wall cylinders in compression is a classical structural analysis problem, which is relevant to many industries and applications such as the design of space launch vehicles. It is well known that experimental buckling loads are much lower than predicted by classical analytical methods. This has been attributed to the presence of imperfections in real cylinders, such as imperfections in geometry, load application and material.

Current industry approaches for analysis and design of cylindrical launch vehicles and other structures rely on empirical guidelines from NASA in 1968 (SP-8007) [1], which propose a knockdown factor for theoretical buckling load predictions. However, these are generally very conservative, and have been largely determined from experimental data for metallic cylinders. This factor can be inappropriate for buckling of cylinders manufactured using composite materials, where for example two identical cylinders with different stacking sequences can have very different buckling behaviour [2]. As such, there is a need for an analysis approach suitable for industrial application that takes the properties of composite materials into account.

With regards to sandwich materials, NASA also published guidelines for buckling of sandwich structures, which was based on less experimental data, and all involved isotropic facesheets [3]. No other guidelines have been published since regarding knockdown factors

for sandwich structures, particularly with facesheets made using composite material. There have been numerous published studies that have highlighted the differences between behaviour of monolithic and sandwich materials, and between isotropic and composite sandwich materials (see, for example, recent review articles [4, 5]). In the meantime, sandwich materials are commonly used on lightweight structures such as space launcher vehicles, which commonly have compression as a critical design case. As such, there is a need for any new buckling analysis technique to incorporate and analyse the effect of using sandwich laminates.

Various other approaches have been adopted for predicting realistic buckling loads, though these have limitations, particularly within the context of a design process. The incorporation of geometric imperfections using the structural eigenmodes is a commonly applied technique [6]. Alternatively, the use of a database of known imperfection patterns has been proposed as a means of assessing the sensitivity of a design using numerical analysis [7]. Other approaches involve taking into account non-traditional imperfections such as those involving the loading or material [8], the use of statistics-based techniques [9, 10], or progressive damage modelling [11, 12]. However, for these approaches, the appropriate parameters to apply are often not known *a priori* for a given structure and manufacturing process, so that experimental testing would be required for validation. This is expensive, and can become impractical in a design context, particularly where designs are being investigated that are outside of the range of those previously considered. This highlights the need for an analysis methodology to be developed that considers the

practicalities of industrial context, where rapid analysis is required for initial design studies without the need for experimental validation.

In response, an approach has been developed that is known as the Single Perturbation Load Analysis (SPLA) [13]. In this approach, a lateral load is first applied to the cylinder, and this load is maintained throughout the application of axial compression leading to buckling. This analysis assumes that a single lateral buckle is a worst-case, realistic and stimulating imperfection pattern for a cylinder in compression. The lateral load causes a knockdown in the buckling load of the cylinder. It was found that small perturbation loads cause a roughly linear reduction in buckling load up to a certain transition point, after which increasing perturbation loads cause only a very small reduction relative to the transition point. This transition point was then proposed as a design point, and shown to provide buckling predictions that were less conservative and more appropriate than those of the benchmark NASA guidelines [13].

In addition to the benefits in conservatism, the use of a technique that is based on a finite element (FE) model has the advantage of being able to account for the exact behaviour of the material and structure. This allows for the incorporation of the effect of composite materials and sandwich laminates with any combination of lay-up and core materials, as part of the analysis of structure with any level of complexity. Further, the use of a technique that is able to identify a design point from only a series of numerical analysis results is highly advantageous within an industrial design context. In this way, the SPLA does not require measuring real imperfections, or information with regards to the manufacturing technique or loading imperfections, or any other information that would

require manufacture or experimental testing. Such a technique has the potential to allow for the development of guidelines for imperfection-sensitive composite and sandwich structures, which is the topic of ongoing research [14, 15].

Despite the success of the previous work with the SPLA approach, the applicability of the technique in different contexts and for different structures has not been demonstrated. In their work, Huhne et al. only reported results for cylinders of a fixed geometry, with the same radius and thickness, although different layups were studied [13]. The results for the SPLA were also presented in terms of buckling load against perturbation load. However, it is unclear if the SPLA is applicable to cylinders of different geometry, how the SPLA applies to cylinders made from sandwich laminates and how best to apply the SPLA in a design context for cylinders of different thickness. It is also unclear how the SPLA compares to analysis of imperfections using eigenmodes, and how the SPLA applies to cylinders with small and large cutouts.

In this work, FE analysis is used to investigate the SPLA technique for different cylinder configurations. The SPLA is applied to cylinders of monolithic composite laminate and sandwich laminates of varying thickness. **This requires a modification to the way in which the SPLA results are considered, which considerably improves the generality of the technique.** The results from the SPLA investigation are compared to an eigenmode-based analysis, in terms of the imperfection sensitivity and practicality in a design context. **Comparison of the SPLA approach with eigenmode-based analysis has not previously been presented in literature.** Cylinders with small and large cutouts are then investigated with the SPLA approach and the interaction between the perturbation and the cutout assessed. **The**

results provide the first application of the SPLA technique to sandwich laminates and cylinders with cutouts. This extends the understanding of both the imperfection sensitivity of monolithic and sandwich cylinders with and without cutouts, and of the applicability and usefulness of the SPLA technique.

2. Cylinder definition

A nominal cylinder geometry was defined, which was based on previous experimental and numerical investigations by one of the authors [16-18]. The cylinder geometry is illustrated in Figure 1, and is based on using a free length (L) of 540 mm and a radius (R) of 350 mm. Monolithic laminates were defined that were based on a carbon/epoxy uni-directional tape IM7/8552, with material properties taken from literature [19, 20] and given in Table 1, where E , G , ν are the stiffness modulus, shear modulus and Poisson's ratio using the fibre (1) and in-plane transverse (2) directions, ρ is the density, t_{ply} is the ply thickness, X is ply strength in the fibre direction in tension (T) and compression (C), Y is ply strengths in the in-plane transverse direction, and S_L is the in-plane shear strength. Sandwich laminates were defined that used the same carbon/epoxy material and incorporated a central section of ROHACELL[®] closed cell foam 71WF, with material properties taken from the material supplier [21] and given in Table 2, where $\sigma_{max,T}$ is the maximum tensile strength. The composite and core materials were selected as they are representative of materials used on aerospace structures.

Two different monolithic laminates were defined: a “quasi-isotropic” $[0,90,45,-45]_s$ laminate and a “cross-ply” $[0,90,90,0,90]_s$ laminate. Two different sandwich configurations

were studied, which all involved two-ply [0,90] and [90,0] facesheets and varying core thickness, where a “thin” core of 2 mm and a “thick” core of 8 mm was applied. As such, the total shell thickness, t , was 1.0 mm for the quasi-isotropic laminate, 1.25 mm for the cross-ply laminate, 2.5 mm for the thin sandwich laminate and 8.5 mm for the thick sandwich laminate. The quasi-isotropic laminate was selected as it is a typical laminate used in aerospace structures. The thin core sandwich laminate was chosen as it had comparable buckling load to the quasi-isotropic laminate. The thick core sandwich laminate was used as it was the thickest core possible such that buckling occurred before material failure. The cross-ply laminate was investigated as it provided a similar buckling mode to the sandwich panels, with a different shell thickness but similar buckling load to the quasi-isotropic laminate.

3. Perturbation Analysis

A finite element model was generated of the cylinder for analysis in Abaqus/Explicit 6.8-1. The model used 4-node reduced integration thick shell elements (S4R) [22]. The mesh used was based on element dimensions of 10 mm $\times$ 10 mm, which involved 220 elements around the cylinder circumference and 54 elements in the cylinder axial direction. Analyses were run on a 32-bit Intel Core 2 Duo 2.25 GHz CPU processor.

The modelling and analysis strategy applied in this work was based on the analysis reported by one of the authors previously in Ref. [17]. In that work, extensive investigations into suitable modelling parameters were conducted, and validation was conducted in comparison with experimental results. This is also similar to numerical investigations presented by other authors [8, 9, 11, 13], which similarly demonstrated close

comparison between detailed shell finite element models and both analytical and experimental results. Furthermore, the mesh density, element type and solver type were verified as appropriate for this work using preliminary sensitivity studies in comparison with analytical models.

The boundary conditions applied are illustrated in Figure 1, and were applied in two analysis steps. In the first analysis step, the perturbation load (F) was applied, with both ends fixed in displacement. The perturbation load was applied such that it corresponded to a constant loading rate of approximately 100 mm/s. As the amount of displacement varied for different perturbation loads, initial trials were run for each analysis in order to determine an appropriate loading rate in terms of the corresponding lateral displacement δ . In the second loading step, the perturbation load F was held constant, whilst a compressive displacement λ was applied with a constant loading rate of 10 mm/s and the compressive load P recorded. The buckling load was taken as the maximum load, $P_{\max}$, experienced by the cylinder, where global buckling or “snap-through” was linked to the loss of load-carrying capability in all cases. The loading rates for the first and second loading steps were verified as appropriate in preliminary studies, which balanced the need for computational efficiency with accuracy and minimisation of dynamic effects at higher loading rates. The analysis did not consider local buckling, for example buckling of the individual sandwich facesheets.

Material properties were considered to be linear elastic, and material failure was not considered as part of determining the panel maximum load. This is because the focus of the investigation was on buckling loads and imperfection sensitivity, and gaining insight

into the SPLA technique. More detailed design studies or analysis in comparison with experimental results should consider material failure, and ideally capture the progression of all critical damage modes. This has previously been demonstrated by the authors [11, 12], where accurate predictions and detailed design studies of composite postbuckling panels were achieved using a progressive damage analysis methodology.

The results of the perturbation analysis for all of the cylinders investigated is summarised in Figure 2, which presents the buckling load for increasing perturbation load. In previous work, SPLA results were presented as buckling load versus perturbation load [13]. In this work, it is proposed that SPLA results are more suitably presented by normalising the buckling load using the buckling load with no perturbation applied, P_0 . Further, it is proposed that instead of presenting results in terms of perturbation load, F , that the lateral displacement, δ , which results from applying the perturbation load is used. In this respect the perturbation is still controlled and applied using the load, but the resultant lateral displacement presents a more useful output value. This displacement is presented by normalising with respect to the total shell thickness, t . The presentation of SPLA results using normalised load versus normalised lateral displacement was found to produce results that were more suitable for comparison across different shell thickness, and more meaningful from both a comparative analysis and design point of view. This is demonstrated and discussed in the following sections.

The SPLA results in Figure 2 show that for all panel configurations, introduction of a perturbation caused a linear reduction in buckling load until a transition point, after which increasing perturbation caused almost no further reduction in buckling load. This bi-linear

behaviour was far more pronounced for the monolithic laminates than for the sandwich panels. Comparing the two monolithic laminates, the results showed that the cylinder with the quasi-isotropic laminate was more sensitive to imperfections, as the perturbations caused larger knockdowns in the buckling load. Despite this, the transition point occurred at the same value for both laminates, which corresponded to a perturbation displacement equal to half the shell thickness ($\delta / t = 0.5$). The perturbation load at the transition was different for the different laminates, which illustrates the benefits in considering the applied perturbation in terms of the normalised lateral displacement.

Figure 3 shows radial displacement patterns for different perturbations in comparison with the eigenmode. The buckling pattern was taken from just after the cylinder reached maximum load, and is considered the pattern most indicative of the buckling mode at global buckling onset. All buckling patterns are presented in the XZ plane, with axial compression in the vertical (Z) direction. The “small” perturbation ($\delta / t = 0.25$) corresponds to the region of linearly decreasing buckling load with perturbation, and the “large” perturbation ($\delta / t = 1.0$) corresponds to the region of negligible reduction in buckling load with increasing perturbation. From these results, it can be seen that small perturbations act only to trigger the naturally occurring buckling pattern, as the buckling pattern is similar to that of the eigenmode. In contrast the large perturbation causes a buckling mode that is different than the eigenmode, so it can be considered that the perturbation is having an unrealistically large influence on the buckling pattern. These conclusions mirror those that were found previously [13]. However, the presentation of these results in terms of normalised perturbation displacement is significant, as it can be

seen that the interpretation of “small” and “large” perturbation displacements relative to the shell thickness is common across different laminates.

The results for the quasi-isotropic monolithic laminate show anomalous behaviour at the transition point. For this laminate, the buckling load at transition appears too low, as it is noticeably lower than the values at higher perturbation displacements. Further investigation of the buckling behaviour at perturbation displacements around the transition point revealed that a local buckling mode appeared at the site of the perturbation displacement before the onset of global buckling. The local buckling mode was seen as a large local increase in displacement, before the global snap-through and loss of load-carrying capability. This local buckle occurred at the location of the perturbation load. This is illustrated in Figure 4 for $\delta / t = 0.6$, which shows the occurrence of local buckling at $P / P_0 = 0.51$ (101 kN) in comparison with global buckling at $P / P_0 = 0.63$ (123 kN). Similar local buckling was also seen for $\delta / t = 0.73$ and 1.0. However, for $\delta / t = 0.5$, no local buckling was seen, but the buckling load is noticeably lower than all cases with increased perturbation. From these results it is deduced that the local buckling mode interacted with the global buckling mode to trigger the global snap-through of the cylinder. This behaviour is unrepresentative of the other results, and demonstrates that the perturbation load triggered unreasonable and anomalous behaviour in this specific case.

Following this identification of anomalous behaviour around the transition point, it is proposed for design purposes that a more suitable perturbation would correspond to a value of $\delta / t = 1.0$, or $\delta = 1t$. That is, in the context of a design analysis, it is proposed that a

suitable perturbation to apply that allows for a reasonable estimate of the knockdown factor from imperfections would be one that corresponds to a displacement equal to the shell thickness. This design point has the value of avoiding the potential for anomalous behaviour around the transition point, and is practical from a design standpoint.

For the sandwich laminates, the results of the SPLA in Figure 2 show that the bi-linear behaviour was less clear, though still evident. The sandwich laminates were less sensitive to imperfections than the monolithic laminates, as the buckling knockdown factors were higher. Increasing the core thickness was seen to make the panel less imperfection-sensitive (higher buckling factors), and smooth out the bi-linear behaviour to a small degree. Despite this, the sandwich laminates still demonstrated that a perturbation displacement of $0.5t$ corresponded to the transition point, such that a perturbation displacement of $1t$ remained a suitable design point.

Figure 3 presents the buckling patterns for the thin and thick core sections for varying perturbation displacement. From this, it can be seen that both sandwich laminates showed similar behaviour to the monolithic laminates in terms of the effect of small and large perturbation displacements. However, as the core thickness increased, the number of half-waves in the buckling pattern decreased – in other words the size of the buckling waves increased. This effect was such that the buckling pattern with any perturbation involved one half-wave of the order of the half-wave in the eigenmode, and smaller half-waves on either side. This behaviour indicates that the buckling pattern was influenced by the overall cylinder length, such that a complete set of “natural” buckling waves similar to the eigenmode could not properly develop. The location of the applied perturbation load in the

axial (Z) direction was also studied, and found to have negligible influence on the buckling knockdown factor. These results can be used to explain the lack of clear bi-linear behaviour in the SPLA results in Figure 2, and is an important outcome for cylinder design where the buckling half-wave length is comparable to the overall cylinder length.

An investigation was conducted into the onset of material failure, in comparison with the application of the perturbation displacement. The analysis of material failure used the material strengths for the composite and core material given in Table 1 and Table 2, and a simple maximum strength approach. For the composite laminates, stresses were analysed in each ply in the ply coordinate system. From this investigation, no mode of material failure occurred for the monolithic laminates or the thin sandwich laminate, considering applied perturbation displacements up to $2t$. However, for the thick sandwich laminate, in-plane shear failure and transverse tension failure in the 0° plies were predicted to occur at perturbation displacements around $1t$. Further, tension failure in the core and fibre failure in a 0° ply were predicted to occur at perturbation displacements of around $1.8t$ and $2t$ respectively. This highlights the need to consider the effect of the perturbation displacement on the material prior to experimental testing, and also illustrates a limitation of the SPLA technique with respect to thick laminates and large perturbations.

4. Eigenmode Analysis

The cylinder of the previous section was analysed using eigenmode-based analysis, for comparison with the SPLA. All monolithic and sandwich laminate configurations previously described were considered. The lowest buckling mode was first found, and this was used to impose an imperfection pattern on the nominal cylinder geometry. The

imperfection pattern was measured based on the ratio of the maximum displacement magnitude to the shell thickness, a / t , and a range of a / t values were investigated. The cylinders with eigenmode-based imperfections were then loaded in axial compression as in the previous analysis, and the buckling load and load history recorded as output.

The results of the investigation are shown in Figure 5 and Figure 6, where Figure 5 presents the normalised buckling load as a function of imperfection magnitude a / t , and Figure 6 presents the load history for the cross-ply cylinder with different imperfection magnitudes. The results in Figure 5 show that increasing the imperfection magnitude led to reductions in the buckling load across all panels. Similar to the SPLA results, the eigenmode results show a rapid decrease in buckling load for small imperfections, leading to a more gradual or even negligible reduction for large imperfections. In this respect, the eigenmode analysis agrees in character with the SPLA. Further, the relative imperfection sensitivity of each cylinder is in agreement between the two approaches, where the quasi-isotropic laminate shows the largest reductions in buckling load. However, the extent of the buckling load knockdowns is significantly higher in the eigenmode analysis, where for example the quasi-isotropic laminate showed a knockdown factor around 0.40 for large imperfections but around 0.60 for large perturbation displacements.

The load history results in Figure 6 show the effect of increasing imperfection magnitude for the cross-ply laminate. These results show that the eigenmode approach reduces the pre-buckling stiffness of the cylinder, and leads to infeasible behaviour for large imperfections. This infeasible behaviour is seen as a change from the sharp snap-through behaviour at global buckling with clear loss in load-carrying capability, to a more

gradual buckling mode shape change that is not representative of the nominal case or real experimental cylinder results. This suggests that for this laminate the eigenmode approach is only suitable for small imperfections magnitudes less than around $0.5t$. This is confirmed by the knockdown factor results in Figure 5, where the results for the cross-ply laminate show an increase in buckling load for large imperfection magnitudes. This is in contrast to the behaviour seen by the other panels of a continuously decreasing trend of buckling factor. This observation is important in comparison with the SPLA, as it indicates that the eigenmode analysis can introduce infeasible behaviour for some laminates, with the amount of imperfection required to generate this behaviour being inconsistent for different laminates. Additionally, unlike the SPLA results, the eigenmode analysis results do not present a clear design point for a suitable imperfection that is common across all laminates. These observations highlight the advantages of the SPLA technique over the use of eigenmodes, particularly within a design context.

5. Perturbation Analysis with Panel Cutouts

An investigation was conducted into the application of the SPLA to cylinders with cutouts. A “large” cutout of 180×180 mm and a “small” cutout of 25×25 mm were considered. The large cutout was selected as it represented a significant portion of the cylinder length, and corresponded to a significant structural feature such as windows or access panels found on aerospace structures. The small cutout was selected as it represented a minor structural feature, with a size similar to the damage detection limit specified for some aerospace applications. Both cutouts were square and located half-way along the axial length of the cylinder. Reinforcements around the cutouts were not considered. The

analysis procedure outlined previously was applied, where only the global buckling response was considered, and material failure at the hole edge was ignored. In this respect, the choice of a square cutout with sharp corners was suitable and practical. The cutouts were created by deleting elements, where the large cutout is shown in Figure 7, and the small cutout required a local mesh refinement from a 10×10 mm mesh to a 5×5 mm mesh before element deletion, as shown in Figure 8.

For both large and small cutouts, the location of the applied perturbation load was found to influence the cylinder buckling in a way not seen for the intact cylinders. As such, a range of perturbation locations were investigated, in order to find the location providing the “worst” or lowest buckling knockdown factor. As part of this investigation, it was found that the influence of the location could be related to the buckling pattern of the cylinder with cutout but without perturbation loads. The buckling patterns of the different configurations are shown in Figure 9, where the buckling pattern is the radial displacement just after buckling, and illustrates the buckling mode that triggers the global snap-through behaviour. Locations where buckling waves first appeared were identified, and used to study whether these were “stimulating” and worst-case perturbation locations. This aspect was complicated by the rapid movement of buckling waves during the global snap-through process and the multiplicity of buckling waves, such that the worst location was not obvious from inspection, and many different locations were considered.

To investigate the perturbation location, a perturbation load was applied such that it produced a perturbation displacement of $1t$. The amount of load required to produce $1t$ of displacement varied with location, so that the appropriate perturbation load was first

determined using a preliminary analysis for each location. Once the worst perturbation load was identified, it was used to study the buckling knockdown factor for a range of δ / t values.

Table 3 summarises the effect of introducing the cutouts, in terms of the buckling load relative to the intact cylinder, with no perturbation load. From these results, the sensitivity of the cylinders to the cutout varied, and agreed with the relative imperfection sensitivity identified previously from the perturbation analysis. The quasi-isotropic laminate had the largest reduction in buckling load, showing a 57% reduction with a large cutout and 70% reduction for the small cutout. The sandwich laminates were the least sensitive to the cutout, even for the thin sandwich laminate where the total shell thickness was comparable to the monolithic laminates.

In comparison with the perturbation results presented previously, it can be seen that the effect of introducing a large cutout is similar to introducing a large perturbation (around $\delta = 1t$). Similarly, the effect of introducing a small cutout is similar to that of a small perturbation (around $\delta = 0.25t$). This analogy provides close comparison across both cutout sizes and all laminates, though shows slightly less applicability for the sandwich laminates where the large perturbations had a less severe effect than the large cutouts. Despite this, the analogy is very useful in characterising the triggering effect of both perturbation load and cutout, and as a guideline with application to the design process.

For cylinders with a large cutout and perturbation loads, Table 4 summarises the results of the perturbation location investigation, which shows the buckling knockdown

factors for each location for a perturbation of $1t$. For this study, the locations are summarised in Figure 10, where with reference to Table 4 the “first buckling corner” (F) and “first buckling mid” (G) were locations at which a buckle appeared around the point at which global snap-through initiated. With reference to Figure 9, Location F was relevant to the quasi-isotropic laminates and thin sandwich laminates, and location G was relevant for the cross-ply and sandwich laminates.

The results in Table 4 show that for all laminates with large cutouts, the critical perturbation location was the “far-field” location, diametrically opposite to the cutout. For the thick sandwich laminate, there was a larger knockdown when the perturbation location was offset from the mid-length position to the “lower” position (B). This was not seen for the other laminates, which indicates that for the thick sandwich core the large buckling waves relative to the cylinder length affected the critical perturbation location. This is similar to behaviour previously seen, where the interaction between the large buckles and cylinder length produced atypical perturbation response.

For all cylinders with large cutouts, it was seen that the application of perturbation at the far-field location led to the global snap-through being triggered at both the far-field and cutout locations. Using the analogy identified previously, this is similar to the cylinder having two perturbation loads applied on either side of the cylinder simultaneously. In contrast for perturbation loads at the cutout edge, the global snap-through was only triggered from the cutout location. In this respect, the large cutout is a significant structural feature, such that any additional perturbation in the vicinity of this feature was not seen to further reduce the buckling load substantially.

The results of varying the perturbation displacement at the worst location with a large cutout are shown in Figure 11. From these results, the buckling knockdown factors for all cylinders were high, demonstrating that the cylinders with a large cutout were not particularly sensitive to perturbations. This indicates that the introduction of the cutout had already significantly reduced the buckling load, and further application of perturbation loads did not cause significant additional reductions. The results also demonstrate that the sandwich panels were less sensitive to imperfections than the monolithic laminates, even for a comparable shell thickness, which agrees with similar previous observations.

For cylinders with a small cutout and perturbation loads, Table 5 summarises the results of the perturbation location investigation, which shows the buckling knockdown factors for each location for a perturbation of $1t$. For this study, the locations are summarised in Figure 10, where the “first buckle” location was identified as a location close to the first buckling at global snap-through initiation for all laminates. This is illustrated in Figure 9, where the buckling patterns for all laminates are shown. The results in Table 5 show that for all laminates with small cutouts, the critical perturbation location was the “first buckle” location (C). As such, it was concluded that for small cutouts, applying the perturbation at a stimulating or triggering location close to the cutout provided a worst-case perturbation load. This implies that for small cutouts, the interaction between the cutout and a closely located perturbation was more critical than for a cutout with a perturbation in the far-field location. For the thick sandwich laminates, subsequent analysis of the “far-field lower” location of the large cutouts produced a slightly lower knockdown factor (0.92) than the stimulating location (0.93). However, the difference in knockdown

factor was very small, such that it was concluded that the stimulating location remained a suitable worst-case location for all configurations.

The results of varying the perturbation displacement at the worst location with a small cutout are shown in Figure 12. From these results, the buckling knockdown factors for all cylinders were high, demonstrating that as with large cutouts the cylinders with a small cutout were also not particularly sensitive to perturbations. For the monolithic laminates the buckling factors were reduced relative to the large cutout case, but for the sandwich laminates the buckling factors were very similar to the large cutout results. This result suggests that for sandwich laminates the size of the cutout does not influence the perturbation sensitivity, which is an important result for the design of sandwich laminate cylinders.

6. Conclusion

An investigation into the application of the SPLA has been conducted across a range of configurations. The SPLA has been applied to cylinders of different shell thickness and for sandwich laminates. It was found that across configurations the SPLA results are best presented in terms of buckling knockdown factor and perturbation displacement as a ratio of shell thickness. Different orthotropic laminates showed varying sensitivity to perturbations, and sandwich laminates were less sensitive than monolithic laminates to perturbations. Based on the results, a perturbation displacement equal to the shell thickness was proposed as a suitable design point for applying the SPLA to cylinders of any thickness or laminate type. The need to consider material failure for large perturbations and thick laminates was demonstrated, with material failure seen for the thick laminate and

perturbation displacements larger than the shell thickness. The SPLA was found to have advantages over the use of eigenmodes for imperfection analysis, as it provided a clear design point for all laminates, did not produce infeasible behaviour, and did not require experimental data for calibration. The SPLA was studied for cylinders with cutouts, and the effect of small and large cutouts was shown to be analogous to the effect of small and large perturbations. Cylinders with cutouts and perturbations were sensitive to the location of perturbation application, and critical worst-case locations were identified and discussed. Cylinders using sandwich laminates of large core thickness gave results that were at times inconsistent with other laminates, due to the large buckling waves relative to the panel length.

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List of figures

Figure 1: Cylinder geometry and boundary conditions

Figure 2: Perturbation analysis

Figure 3: Buckling patterns (radial displacement).

Figure 4: Radial displacement (mm) at varying applied load, quasi-isotropic laminate, $\delta / t = 0.6$.

Figure 5: Eigenmode analysis results.

Figure 6: Load-displacement for varying eigenmode-based imperfections

Figure 7: FE mesh, cylinder with large cutout, complete cylinder in XZ plane.

Figure 8: FE mesh, cylinder with small cutout, close-up of cutout in XZ plane.

Figure 9: Buckling pattern (radial displacement), cylinders with cutouts but no perturbation.

Figure 10: Perturbation load locations, cylinders with cutouts. Top row: Large cutout. Bottom row: Small cutout.

Figure 11: Perturbation analysis results, cylinders with large cutouts.

Figure 12: Perturbation analysis results, cylinders with small cutouts.

Table 1: Carbon/epoxy unidirectional ply material properties.

Property	Value
E_{11}	150 GPa
E_{22}	9.08 GPa
G_{12}	5.39 GPa
ν_{12}	0.32
ρ	1500 kg/m ³
t_{ply}	0.125 mm
X_T	2323 MPa
X_C	1200 MPa
Y_T	101 MPa
Y_C	200 MPa
S_L	130 MPa

Table 2: Foam core material properties.

Property	Value
E	105 MPa
G	42 MPa
ρ	75 kg/m ³
$\sigma_{\max,T}$	2.2 MPa

Table 3: Buckling knockdown factors from cutouts, with no perturbation applied.

	quasi-isotropic	cross-ply	sandwich, thin	sandwich, thick
large cutout	0.57	0.76	0.82	0.87
small cutout	0.70	0.89	0.96	0.96

Table 4: Cylinders with large cutouts, buckling knockdown factor for different perturbation locations.

location	label	quasi-isotropic	cross-ply	sandwich, thin	sandwich, thick
far-field	A	0.94	0.95	0.96	0.98
far-field lower	B	0.95	0.95	0.96	0.92
cutout side	C	0.99	1.00	1.00	1.00
cutout corner	D	1.00	1.01	1.00	0.97
cutout bottom	E	1.00	1.00	1.00	0.99
first buckle corner	F	0.97	0.99	1.00	0.96
first buckle mid	G	0.96	0.97	0.98	0.97

Table 5: Cylinders with small cutouts, buckling knockdown factor for different perturbation locations.

location	label	quasi-isotropic	cross-ply	sandwich, thin	sandwich, thick
far-field	A	0.90	0.94	0.95	0.97
cutout side	B	0.92	0.96	0.97	0.99
first buckle	C	0.82	0.89	0.94	0.93
cylinder side	D	0.90	0.94	0.94	0.96

Figure 1: Cylinder geometry and boundary conditions.

Figure 2: Perturbation analysis results

Figure 3: Buckling patterns (radial displacement).

Figure 4: Radial displacement (mm) at varying applied load, quasi-isotropic laminate, $\delta / t = 0.6$.

Figure 5: Eigenmode analysis results.

Figure 6: Load-displacement for varying eigenmode-based imperfections, cross-ply laminate.

Figure 7: FE mesh, cylinder with large cutout, complete cylinder in XZ plane.

Figure 8: FE mesh, cylinder with small cutout, close-up of cutout in XZ plane.

Figure 9: Buckling pattern (radial displacement), cylinders with cutouts but no perturbation.

Figure 10: Perturbation load locations, cylinders with cutouts. Top row: Large cutout. Bottom row: Small cutout.

Figure 11: Perturbation analysis results, cylinders with large cutouts.

Figure 12: Perturbation analysis results, cylinders with small cutouts.