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A methodology for hydrocode analysis of ultra-high molecular weight polyethylene composite under ballistic impact

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Abstract

Ballistic performance analysis of ultra-high molecular weight polyethylene (UHMW-PE) is critical for the design of armour systems against ballistic threats. However, no validated modelling strategy has been published in literature for UHMW-PE composite that captures the penetration and damage mechanisms of thick targets impacted between 900 m/s to 2000 m/s. Here we propose a mechanistically-based and extensively validated methodology for the ballistic impact analysis of thick UHMW-PE composite. The methodology uses a non-linear orthotropic continuum model that describes the composite response using a non-linear equation of state (EoS), orthotropic elastic-plastic strength with directional hardening and orthotropic failure criteria. Interlaminar failure is modelled by dividing the panel into sub-laminates connected by breakable bonds. The model is extensively validated using experimental ballistic data for a wide range of UHMW-PE target thicknesses up to 102 mm against 12.7 mm and 20 mm calibre fragment simulating projectiles (FSPs) with impact velocities between 400 and 2000 m/s. Very good overall agreement with experimental results is seen for depth of penetration, ballistic limit and residual velocity, while the penetration mechanisms and target bulge behaviour are accurately predicted. The model can be used to reduce the volume of testing typically required to design and assess thick UHMW-PE composite in ballistic impact applications.

Keywords: Polymer-matrix composites, Ballistic Impact Behaviour, Modelling, Polyethylene fibres

1 Introduction

Composites reinforced with ultra-high molecular weight polyethylene (UHMW-PE) fibres are increasingly being used in protection against ballistic threats due to their high resistance to penetration and low weight. The material shows a mass efficiency against fragment simulating projectiles (FSP) threats of 300% to 500% compared to armour steel and 130% to 160% for polymer matrix composites reinforced with aramid, carbon or glass fibre [1]. Due to the extensive deformation exhibited by UHMW-PE composite under ballistic impact, multiple large targets are required to determine a single ballistic limit velocity. Such testing can be prohibitively expensive when considering a range of thicknesses and armour configurations against multiple ballistic threat types. In order to reduce testing efforts, it is highly desirable to establish computationally efficient numerical models that accurately predict the ballistic response of the material.

Numerical simulation of UHMW-PE composite under ballistic impact presents a significant challenge due to the large difference between the in-plane and interlaminar mechanical properties. The in-plane tensile strength, for example is approximately three orders of magnitude greater than the out-of-plane tensile strength [2]. Such large differences promote numerical instabilities (e.g. zero energy deformation modes or hour glassing) and require complex constitutive models with many parameters to describe the material behaviour at the macro-scale.

Chocron et al. [3] proposed a meso-scale model of UHMW-PE composite that discretises the laminate into strips of fibre bundles. The approach uses a simple linear-elastic orthotropic material model to represent the fibre bundles, and was successful in predicting the response

of UHMW-PE composite strips, layers ([0/90]₂), and targets up to 11.5 mm against 7.62 mm diameter FSPs. However, the computational cost of meso-scale models is very high, making them impractical for thick UHMW-PE composites where a large number of bundle elements would be needed. Other models have been proposed to analyse the ballistic impact response of UHMW-PE composite, including multi-scale [4] and continuum [5] approaches. However, only limited validation of these models for impact velocities below 900 m/s has been performed and no validation has been conducted for higher impact velocities. At higher impact velocities, a non-linear equation of state (EoS) is required to account for the non-linear shock compressibility of the material; this is not captured with these models.

Lässig et al. [2] conducted extensive experimental characterisation of UHMW-PE composite (Dyneema[®] HB26) and derived material parameters for a continuum non-linear orthotropic model (that includes a non-linear EoS) implemented in ANSYS[®] AUTODYN[®] [6–9]. With these parameters, the model was validated for the impact of 15 mm thick UHMW-PE plates by spherical projectiles travelling at hypervelocity (2052 m/s to 6591 m/s). The material dataset was recently used by Nguyen et al. [10] to model thicker UHWE-PE targets (up to 50 mm) when impacted by FSPs at velocities below 2000 m/s. At these impact velocities, material strength becomes increasingly important in controlling the deformation and perforation responses. Results showed significant under-prediction of the ballistic limit due partly to premature through-thickness shear failure. This failure occurs as a result of coupling within the failure model, which is sensitive to the very weak through-thickness tensile properties of the material.

In this paper, a methodology is proposed for modelling the ballistic impact analysis of thick UHMW-PE composites using a hydrocode model that incorporates a non-linear EoS and an orthotropic elastic-plastic material definition with multiple failure modes. The in-plane and out-of-plane failure modes are decoupled in the bulk material through sub-laminate

discretisation of the target. The model is validated against a large amount of experimental ballistic impact data recently reported by Nguyen et al. [11]. Validation includes ballistic limit (V_{50}) predictions for UHMW-PE composite targets up to 102 mm thick against 12.7 mm and 20 mm FSPs, as well as depth of penetration (DoP) of semi-infinite UHMW-PE composite targets against 20 mm FSP. Quantitative validation of the model is also made in terms of the target bulge geometry and apex and their dependence on the FSP velocity profile, and the plastic hinge position of the bulge in the composite.

2 Ballistic Impact Model

2.1 UHMW-PE Material Model

The non-linear orthotropic material model developed in [6–9] and implemented in ANSYS® AUTODYN® is used to model the ballistic impact response of UHMW-PE composite. The material model includes orthotropic coupling of the material volumetric and deviatoric responses, non-linear equation of state, orthotropic hardening, stress-based composite failure criteria, and orthotropic energy-based softening. The material parameters for the UHMW-PE composite that were used are detailed in Table 1. For completeness the model will be briefly described below; more details are given in [2,7–9,12].

Table 1. Non-linear orthotropic material model parameters for UHMW-PE composite (Dyneema® HB26)

EoS: Orthotropic					Strength: Orthotropic Yield				
Parameter	Symbol	Value	Units	Source	Parameter	Symbol	Value	Units	Source
Reference Density	ρ	0.98	g/cm ³	[13]	Plasticity constant 11	A_{11}	0.016	-	-
Young's Modulus 11	E_{11}	3.62×10^6	kPa	[2]	Plasticity constant 22	A_{22}	6×10^{-4}	-	-
Young's Modulus 22	E_{22}	5.11×10^7	kPa	[14]	Plasticity constant 33	A_{33}	6×10^{-4}	-	-
Young's Modulus 33	E_{33}	5.11×10^7	kPa	[14]	Plasticity constant 12	A_{12}	0	-	-
Poisson's Ratio 12	ν_{12}	0.013	-	[2]	Plasticity constant 13	A_{13}	0	-	-
Poisson's Ratio 23	ν_{23}	0	-	[2]	Plasticity constant 23	A_{23}	0	-	-
Poisson's Ratio 31	ν_{31}	0.5	-	[2]	Plasticity constant 44	A_{44}	1	-	-
Shear Modulus 12	G_{12}	2.0×10^6	kPa	a	Plasticity constant 55	A_{55}	1.7	-	-
Shear Modulus 23	G_{23}	1.92×10^5	kPa	a	Plasticity constant 66	A_{66}	1.7	-	-
Shear Modulus 31	G_{31}	2.0×10^6	kPa	a	Eff. Stress #1	$\sigma_{eff\#1}$	1.48×10^3	kPa	-
Volumetric Response: Shock					Eff. Stress #2	$\sigma_{eff\#2}$		kPa	-
Grüneisen coefficient	Γ	1.64	-	[15]	Eff. Stress #3	$\sigma_{eff\#3}$	2.7×10^4	kPa	-
Parameter C1	c_0	3.57×10^3	m/s	-	Eff. Stress #4	$\sigma_{eff\#4}$	4.0×10^4	kPa	-
Parameter S1	s	1.3	-	b	Eff. Stress #5	$\sigma_{eff\#5}$	5.0×10^4	kPa	-
Reference Temperature	T_0	293	k	-	Eff. Stress #6	$\sigma_{eff\#6}$	6.0×10^4	kPa	-

Specific Heat	c	1.85×10 ³	J/kgK	[13]	Eff. Stress #7	σ _{eff#7}	8.0×10 ⁴	kPa	-
Failure: Orthotropic Softening					Eff. Stress #8	σ _{eff#8}	9.8×10 ⁴	kPa	-
Parameter	Symbol	Value	Units	Source	Eff. Stress #9	σ _{eff#9}	2.0×10 ⁵	kPa	-
Tensile Failure Stress 11	S ₁₁	1.01×10 ²⁰	kPa	disabled	Eff. Stress #10	σ _{eff#10}	1.0×10 ⁶	kPa	-
Tensile Failure Stress 22	S ₂₂	1.15×10 ⁶	kPa	[14]	Eff. Plastic Strain #1	ε _{eff#1}	0	-	-
Tensile Failure Stress 33	S ₃₃	1.15×10 ⁶	kPa	[14]	Eff. Plastic Strain #2	ε _{eff#2}	0.01	-	-
Maximum Shear Stress 12	S ₁₂	5.75×10 ⁵	kPa	c	Eff. Plastic Strain #3	ε _{eff#3}	0.1	-	-
Maximum Shear Stress 23	S ₂₃	1.20×10 ⁵	kPa	[14]	Eff. Plastic Strain #4	ε _{eff#4}	0.15	-	-
Maximum Shear Stress 31	S ₃₁	5.75×10 ⁵	kPa	c	Eff. Plastic Strain #5	ε _{eff#5}	0.175	-	-
Fracture Energy 11	G _{11C}	790	J/m ²	[2]	Eff. Plastic Strain #6	ε _{eff#6}	0.19	-	-
Fracture Energy 22	G _{22C}	30	J/m ²	d	Eff. Plastic Strain #7	ε _{eff#7}	0.2	-	-
Fracture Energy 33	G _{33C}	30	J/m ²	d	Eff. Plastic Strain #8	ε _{eff#8}	0.205	-	-
Fracture Energy 12	G _{12C}	1.46×10 ³	J/m ²	d	Eff. Plastic Strain #9	ε _{eff#9}	0.21	-	-
Fracture Energy 23	G _{23C}	1.46×10 ³	J/m ²	d	Eff. Plastic Strain #10	ε _{eff#10}	0.215	-	-
Fracture Energy 31	G _{31C}	1.46×10 ³	J/m ²	d	Bonds: Sub-laminate Interface				
Damage Coupling Coefficient	C	0	-	disabled	Parameter	Symbol	Value	Units	Source
					Normal Strength	S _N	5.35×10 ³	kPa	[2,16]
					Shear Strength	S _S	7.85×10 ³	kPa	[3]

^a The plasticity model does not allow stress-strain curve slopes to exceed the elastic modulus value. Under shear there is exponential rise due to fibre realignment that is greater than the measured elastic shear modulus. Since the elastic portion of the curve is small, the modulus associated with the fibre realignment phase is chosen to better match the entire stress-strain curve.

^b Empirically adjusted to match flyer plate impact tests [2]

^c Maximum shear stress calculated from the principal stress theory based on a specimen under uniaxial in-plane tension

^d Cross-woven Kevlar Epoxy composite [7]

2.1.1 Equation of State

The thermodynamic (EoS) response of a material and its ability to carry tensile and shear loads (strength) is typically treated separately within hydrocodes such that the stress tensor can be decomposed into volumetric and deviatoric components [17]. However, anisotropic materials exhibit coupling of these two responses, i.e. hydrostatic stresses lead to deviatoric strains and vice versa. Anderson et al. [18] proposed a constitutive formulation for anisotropic material which allows the use of the theory of shock waves for orthotropic materials with a limited amount of coupling of the volumetric and deviatoric responses in the elastic regime. Here the pressure is composed of the volumetric and deviatoric components, and is defined by:

$$P = P(\varepsilon_{vol}, e) - \frac{1}{3}(C_{11} + C_{21} + C_{31})\varepsilon_{11}^d - \frac{1}{3}(C_{12} + C_{22} + C_{32})\varepsilon_{22}^d - \frac{1}{3}(C_{13} + C_{23} + C_{33})\varepsilon_{33}^d \quad (1)$$

where C are coefficients of the stiffness matrix and $\varepsilon_{11,22,33}^d$ are the deviatoric strains in the principal directions. The pressure contribution from the volumetric strain $P(\varepsilon_{vol}, e)$ is described using the Mie-Grüneisen EoS:

$$P(\varepsilon_{vol}, e) = P_r(v) + \frac{\Gamma(v)}{v} [E - E_r(v)] \quad (2)$$

where v is the volume, e is the internal energy and $\Gamma(v)$ is the Grüneisen coefficient. $P_r(v)$ and $E_r(v)$ refer to a reference pressure and internal energy, respectively. The shock Hugoniot is typically used as a reference condition.

The shock formulation of the Mie-Grüneisen EoS is applied here, where an empirical linear relationship defines the shock and particle velocity relationship:

$$U_s = c_0 + Su_p \quad (3)$$

where U_s is the shock wave velocity, c_0 is the bulk sound speed, S is the slope of the shock-particle velocity curve, and u_p is the particle velocity. The reference pressure, density and internal energy are then calculated from the Rankine-Hugoniot equations. Conditions off the Hugoniot reference curve are approximated with the Grüneisen coefficient (Γ) from the second term in Eq. (2).

In the non-linear orthotropic model, c_0 is calculated from the elastic orthotropic constants and the slope of the $U_s - u_p$ relationship, S , is empirically adjusted to match flyer plate impact test results. The shock response of UHMW-PE composite has previously been shown by Hazell et al. [19] to be similar to that of polyethylene, therefore it is assumed the off Hugoniot response of UHMW-PE composite is also similar. Thus, a Grüneisen coefficient for polyethylene of 1.64 is used in this work [15].

Figure 1 shows measurements by Lässig et al. [2] of the free surface velocity determined by inverse planar plate impact tests on UHMW-PE composite plates. Using the model, numerical results from one-dimensional simulations are also plotted, demonstrating good agreement of the initial and subsequent release waves up to about 3000 ns to 3500 ns post impact. After this time the one-dimensional strain assumption used in the simulation is no

longer valid because the stress waves propagating from the lateral edge of the target specimen affect the free surface velocity measurements. The numerical curves have been produced through empirical adjustment of the S parameter, thereby describing correctly the non-linear shock compressibility.

Figure 1. Free surface velocity trace from inverse plate impact test and numerical validation.

2.1.2 Strength Model

The quadratic yield surface proposed by Chen et al. [20] is used to describe non-linear, irreversible hardening of the material:

$$f(\sigma_{ij}) = a_{11}\sigma_{11}^2 + a_{22}\sigma_{22}^2 + a_{33}\sigma_{33}^2 + 2a_{12}\sigma_{11}\sigma_{22} + 2a_{23}\sigma_{22}\sigma_{33} + 2a_{13}\sigma_{11}\sigma_{33} + 2a_{44}\sigma_{23}^2 + 2a_{55}\sigma_{31}^2 + 2a_{66}\sigma_{12}^2 = k \quad (4)$$

where the nine plasticity coefficients, a_{ij} , represent the degree of plastic anisotropy of the material, σ_{ij} are stresses in the principal material directions, and k is a state variable that defines the current limit of the yield surface. To describe strain hardening, k is replaced by a master effective stress-effective plastic strain curve, defined by 10 piecewise points. It allows the determination of the stress states in any orthotropic direction from the plasticity

coefficients a_{ij} . The effective stress $\bar{\sigma}$ and effective plastic strain $\bar{\epsilon}^p$ in the normal direction is defined by:

$$\bar{\sigma} = \sigma_{ii} \sqrt{\frac{3a_{ii}}{2}} \quad \text{and} \quad \bar{\epsilon}^p = \epsilon_{ii} \sqrt{\frac{2}{3a_{ii}}} \quad (5)$$

and the shear direction by:

$$\bar{\sigma} = \sigma_{ij} \sqrt{3a_{ij}} \quad \text{and} \quad \bar{\epsilon}^p = \frac{\epsilon_{ij}}{\sqrt{3a_{ij}}} \quad (6)$$

The master effective stress-effective plastic strain curve is best defined in the plane that experiences the largest non-linear plastic deformation. In the present work the in-plane shear stress-strain curve is used, and the corresponding plasticity coefficient a_{11} is set to one. The other plasticity coefficients are set where possible to match experimental stress-strain curves for the other orthotropic directions.

2.1.3 Failure Model

Failure in the non-linear orthotropic model is based on a combined stress criterion and is initiated when:

$$\left(\frac{\sigma_{ii}}{S_{ii}(1-D_{ii})} \right)^2 + \left(\frac{\sigma_{ij}}{S_{ij}(1-D_{ij})} \right)^2 + \left(\frac{\sigma_{ki}}{S_{ki}(1-D_{ki})} \right)^2 \geq 1 \quad \text{for } i, j, k = 1, 2, 3 \quad (7)$$

where S_{ii} is the failure strength of the material in the respective directions. The damage parameter, D_{ii} , follows a linear relationship with stress and strain and is defined by:

$$D_{ii} = \frac{L\sigma_{ii,f}\epsilon_{cr}}{2G_{ii,f}} \quad (8)$$

where L is the characteristic cell length, ϵ_{cr} is the crack strain (strain above the failure initiation strain) and $G_{ii,f}$, the fracture energy in the direction of damage.

Figure 2. Stress-strain curves from single element simulations compared to experiments. (a) in-plane tension, (b) in-plane shear, (c) out-of-plane shear and (d) out-of-plane compression.

The strength and failure model is verified against mechanical test results for UHMW-PE composite (Dyneema[®] HB26) under uniaxial tension, compression and shear from a number of different sources [2,3,14,21]. The numerical verification involves simulation of a single element under the appropriate loading and boundary conditions; the results of which are shown in Figure 2. The numerical model is shown to provide good agreement for the in-plane tension and reasonable agreement for the out-of-plane compression simulation. The in-plane shear simulation is also in good agreement; however failure occurs at a higher stress in the numerical model. The experimental curve is from a 45° in-plane tension test, for which failure measurements are known to be dependent to specimen geometry [22]. For example, Lässig et

al. [21] measured failure strengths between 35 MPa and 55 MPa for an UHMW-PE specimen with a 20 mm gauge width tested at different loading rates. Experiments performed by Heisserer [14] on the same material with a gauge width of 80 mm gave failure stresses greater than 120 MPa. Since validation of the model is ultimately performed on ballistic panels with lateral dimensions greater than 300 mm, the larger value is taken.

The agreement for out-of-plane shear stress-strain is not as good, yet still reasonable (Figure 2(c)). This is due to the simplification of the strength model, which describes all stress states in the principal directions as scalable with respect to the master effective stress-effective plastic strain curve by one single plasticity constant per loading mode. In through-thickness shear experiments conducted by Lässig et al. [2], measurements were not made for shear strains greater than 0.4, at which point the specimen had not completely failed. At high shear strains fibre realignment occurs, resulting in a significant increase in the slope (as seen in [2]). This behaviour is shown to be replicated in the model. Under this loading condition complete failure occurs due to either fibre tensile failure as a result of fibre realignment, or fibre shearing. As such, the principal stress is used to determine the maximum out-of-plane failure shear stress for the specimen under uniaxial in-plane tension:

$$S_{12,31} = \frac{S_{22,33}}{2} \quad (9)$$

This provides a value of 575 MPa, which is similar to 560 MPa that was used successfully by Nguyen et al. [11] in an analytical model to predict the ballistic penetration of thick UHMW-PE composite.

2.2 Target Erosion Model

When modelling ballistic impact events on isotropic materials using Lagrangian discretisation, large element distortion occurs that affects time step and numerical stability.

Erosion models based on effective strain have been used to successfully overcome this issue [23], where the effective strain is defined as [24]:

$$\varepsilon_{eff} = \frac{2}{3} \left[\left(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2 \right) - \left(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}\varepsilon_{33} + \varepsilon_{33}\varepsilon_{11} \right) + 3 \left(\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2 \right) \right]^{1/2} \quad (10)$$

However, such formulation is inappropriate for anisotropic materials because the failure strains can be substantially different in different directions due to the directionally-dependant nature of fibre-reinforced composites. When failure occurs in one direction, the corresponding stiffness in that direction is set to zero. This leads to large strains in that direction, which in the case of impact and penetration can be as excessive as the driving force of the impactor continues to expand the element. If the strains in the other directions remain small, then the effective strain does not capture these extreme strains as the calculation has an averaging effect. Furthermore, due to the continuum definition, as the strains increase significantly in one direction following failure, the overall volume of the element increases. This has the effect of artificially preventing failure in the other directions, as forces in these directions generate lower stresses due to the increased volume. This effect becomes more pronounced at the excessive strains seen in penetration simulations.

A new damage-based erosion model was written in a user subroutine to address this problem. In this model, elements are eroded only when they are fully damaged in the in-plane (fibre) direction, which is defined by:

$$\begin{aligned} D_{22} &= 1 \\ D_{33} &= 1 \end{aligned} \quad (11)$$

Failure of elements in the fibre direction also leads to a loss of strength in the associated shear directions, so the above criterion also accounts for elements failing under shear. An out-of-plane criterion (e.g. in the 11 direction) is not included because out-of-plane failure, as will be discussed in the following section, is modelled between the sub-laminate interfaces

through bonded contacts. A global instantaneous erosion strain of 150% was additionally applied to delete any highly distorted elements which affect the time step and numerical stability without playing a further role on the target interaction.

Figure 3 shows the velocity profile of simulations computed using the effective strain criterion (IGS, or instantaneous geometric strain in AUTODYN[®]) with an effective strain of 250% (IGS2.5) and 150% (IGS1.5), in comparison with simulation results using the damage-based criterion (DMG). Cross-sections from the IGS2.5 numerical analysis at two points, A and B, are shown with contours of in-plane material damage. Experimental results are also shown, where the experimental residual velocity and perforation time are indicated together with shaded regions corresponding to the typical experimental variance. The results show that using the IGS erosion model with an effective strain of 250%, at 30 μ s after impact (point A), the model has predicted complete failure through the thickness of the target. However, the projectile velocity continues to reduce beyond this point in time. At 200 μ s after impact (point B), the failed elements have elongated significantly but the projectile velocity still continues to decrease. In reality, this failed material would no longer be influencing the projectile. The unrealistic behaviour is an artefact of the strain-based erosion model for an anisotropic material. Decreasing the effective strain to 150% reduces the extent of this unrealistic behaviour, but does not eliminate it completely. In contrast, the damage-based erosion model gives accurate predictions of the residual velocity and perforation time, in addition to more realistic representations of the material performance as shown at point C in Figure 3.

Figure 3. Projectile residual velocity versus time from impact for 10 mm thick targets impacted by 20 mm FSP at 469 m/s. Right images are cross-section contours through the model mid-plane from the IGS2.5 configuration, corresponding to points on the velocity profiles, where red indicates elements that have failed in the in-plane direction and blue indicates no in-plane failure.

2.3 Target model

The ballistic limit tests performed in [11] used targets measuring 300 mm × 300 mm except for the 76 mm and 102 mm thick panels which had lateral dimensions of 400 mm × 400 mm. The full target is modelled (i.e. no symmetry) because the material is orthotropic and the projectile is asymmetric. In ANSYS® AUTODYN®, the out-of-plane direction is designated as the 11 material direction and the in-plane directions are the 22 and 33 directions. The model used 8-node hexahedral elements and the target was discretised into sub-laminates; to form multiple layers through the laminate thickness. A stack of sub-laminates is generated, with each sub-laminate one element thick, separated by a small gap. This gap was 0.06 mm for targets impacted by 12.7 mm FSP or 0.1 mm for targets impacted by 20 mm FSP to satisfy the master-slave contact (external gap) algorithm used to detect contact between the different bodies. While the addition of these gaps increased the total thickness of the target, the actual thickness of the target was only considered to be occupied by material, i.e the sum

of the thickness of all of the sub-laminates. The sum of all the gaps was small ($<5\%$) compared to the total thickness of the target and was considered to have negligible effects on the ballistic performance.

The element dimension of the target sub-laminates were approximately uniform (aspect ratio of 1) and equal to the projectile mesh size at the impact site. After extending to a radius of one projectile diameter from the penetration zone, the mesh size was radially graded. The mesh size was controlled by the number of elements across the projectile diameter, and the need to maintain the same element size between the projectile and the target penetration zone. Matching the projectile and target mesh is critical in order to avoid a stiffness mismatch due to the relative mesh size of the target and projectile.

Mesh refinement was performed for a DoP test to determine the thickness of the sub-laminate and the size of the elements in the penetration zone. Table 2 shows the cases considered and the resulting number of sub-laminates in the laminate, sub-laminate thickness (also the in-plane element length around penetration zone), the predicted depth of penetration and the percentage difference (% Diff) in depth of penetration relative to the case with the finest mesh. The projectile velocity profile is also important to consider for this mesh refinement study and is shown in Figure 4 for the different cases.

Table 2. Mesh refinement study results on 100 mm DoP target impacted by 20 mm FSP at 1140 m/s.

Elements across projectile diameter	Number of sub-laminates	Sub-laminate thickness (mm)	DoP (mm)	% Diff
6	30	3.3	46.6	-14.5
9	50	2.0	52.0	-4.6
12	60	1.7	53.3	-2.2
15	75	1.3	54.5	0.0

Figure 4. Projectile velocity profile results from mesh refinement study on 100 mm DoP target impacted by 20 mm FSP at 1140 m/s.

In this study, satisfactory convergence was reached for cases where there were 9 or more elements across the projectile diameter, which corresponds to element size around the penetration zone and sub-laminate thickness of 2 mm. This meshing scheme was used for all targets impacted by 20 mm FSP. For targets impacted by the 12.7 mm FSP, 9 elements across the projectile diameter was also used, giving element size around the penetration zone and sub-laminate thickness of 1.4 mm.

The sub-laminates were kinematically joined together using a bonded contact that was breakable through a criterion combining normal and shear stresses:

$$\left(\frac{\sigma_N}{S_N} \right)^a + \left(\frac{\sigma_S}{S_S} \right)^b \geq 1 \quad (12)$$

where σ and S are stress and strength values, subscripts N and S are the normal and shear directions respectively, and the exponents a and b were assumed to be 1.0 due to the absence of combined loading data. No high strain rate experiments have been reported for

UHMW-PE composite, which would allow a determination of the through-thickness tensile and shear strengths at ballistic conditions. These properties are dominated by the matrix material, which is both highly strain-rate and pressure-dependent [3,25].

Dynamic spallation tests performed by Riedel et al. [16] on carbon and aramid fibre reinforced composites have shown through-thickness tensile strength to be between 2 to 5 times higher than quasi-static values [16]. Based on this, the quasi-static through-thickness tensile strength value for UHMW-PE composite (1.07 MPa [2]) was multiplied by a factor of 5, to determine an approximate high strain rate value. The interlaminar shear strength of UHMW-PE composite was characterised in [3] at 500 MPa of hydrostatic pressure. The high pressure value determined is 3 times higher than the value under atmospheric conditions, as the initiation and propagation of cracks are inhibited under hydrostatic pressure [26]. The high pressure interlaminar strength was used in this work, as it more closely approximates ballistic conditions where high pressure propagates through the material upon impact.

No boundary conditions were imposed on the target because high speed video of ballistic impact tests typically showed clamp slippage upon impact due to the low friction coefficient of UHMW-PE composite [11].

2.4 Projectile model

The FSP is made from 4340H steel with a Rockwell C hardness of 30 (MIL-DTL-46593B [27]). The material was described using the Johnson-Cook strength model [28] and the shock formulation of the Mie-Grüneisen EoS, the parameters for which are shown in Table 3.

Table 3. FSP material parameters

EoS: Shock				
Parameter	Symbol	Value	Units	Source
Density	ρ	7.88	g/cm^3	[29]
Gruneisen coefficient	Γ	2.17	-	[15]
Parameter C1	c_0	4.57×10^3	m/s	[15]
Parameter S1	s	1.49	-	[15]
Reference Temperature	T_0	300	k	-
Specific Heat		477	J/kgK	[28]

Strength: Johnson-Cook				
Parameter	Symbol	Value	Units	Source
Shear Modulus	G	7.78×10^7	kPa	[29]*
Yield Stress	A	1.03×10^7	kPa	[29]
Hardening Constant	B	4.77×10^7	kPa	[28]
Hardening Exponent	n	0.18	-	[28]
Strain Rate Constant	C	0.012	-	[28]
Thermal Softening Exponent	m	1.0	-	[28]
Melting Temperature	T_M	1763	K	[28]
Ref. Strain Rate	$\dot{\epsilon}_0$	1.0	s^{-1}	[28]

* calculated from an elastic modulus of 207 GPa and Poisson ratio of 0.33

The Johnson-Cook strength model parameters for 4340H steel were based on those of Steel S-7 [28] with a reduction in the yield strength from 1590 MPa to 1030 MPa to reflect the reported yield strength of 4340H steel [29]. The parameters for strain and strain rate hardening were unchanged. Parameters for the shock EoS of steel were taken from [15]. Simulations with this material model and parameter set showed similar levels of projectile deformation compared to the experiment for the range of impact velocities investigated (Figure 5).

Figure 5. Projectile deformation in the numerical simulation (top) and experiments (bottom).

3 Results & Discussion

3.1 Depth of Penetration of Semi-Infinite Targets

Experimental depth of penetration values for semi-infinite UHMW-PE composite targets against 20 mm FSP [11] are used as a first step to validate and assess the accuracy of the numerical model. In these tests, 100 mm and 150 mm thick 150 mm $\times$ 150 mm targets were impacted by a 20 mm FSP. The targets were backed by a 100 mm thick steel plate to prevent back face deformation of the UHMW-PE target. Experimentally, the DoP was measured from the difference between the original panel thickness and the remaining unperforated thickness after ballistic impact. Numerically the DoP was determined from the sum of all perforated sub-laminates.

Numerical simulations were performed with and without a steel backing, though no discernible difference in penetration depth was seen for the two cases. The DoP results obtained by modelling and experimentation are given in Table 4 and are plotted in Figure 6. Comparison with experiment shows excellent agreement, with both experiment and simulation predicting a linear relationship between DoP and impact velocity.

Table 4. Depth of penetration results

Target Thickness (mm)	Impact Velocity (m/s)	DoP Experiment (mm)	DoP Numerical (mm)	Diff (%)
100	815	30.0	30	0.0
150	991	39.8	40	0.5
150	1038	45.2	44	-2.7
150	1305	61.4	60	-2.3

Figure 6: Depth of penetration results for 20 mm FSP impacts against semi-infinite UHMW-PE composite.

3.2 Ballistic Performance of Finite Thickness Targets

Simulations were conducted to predict the ballistic limit velocity of various thicknesses of UHMW-PE composite panels against 12.7 mm and 20 mm FSP. The number for sub-laminates used to model the target for various target thicknesses is shown in Table 5. The targets were impacted over a range of impact velocities requiring one partial penetration and at least three complete penetrations. The residual velocities of the FSPs following target perforation were fit to the Lambert-Jonas equation [30], Eq.(13), from which an estimate of the ballistic limit was determined:

$$V_R = a(V_I^p - V_{BL}^p)^{1/p} \quad (13)$$

where V_R is the residual velocity, V_I is the impact velocity and V_{BL} is the ballistic limit velocity. V_{BL} , a and p are determined from regression analysis of the impact and residual velocity results. The results of the Lambert-Jonas analysis are shown in Figure 7. Nguyen et al. [11] used high speed photography to measure the projectile residual velocity in experimental ballistic tests on 10 mm and 20 mm UHMW-PE composite targets against

20 mm FSPs . These measurements are plotted in Figure 7 for comparison. The numerical results show excellent agreement with the experiments.

Figure 7: Numerical residual velocity predictions for UHMW-PE composite impacted by (a) 12.7 mm FSP and (b) 20 mm FSP with comparison to experimental measurements. The numerical results are fit to the Lambert-Jonas equation, parameters for which are given in the legend (a, p, V_{BL}).

The numerical ballistic limit predictions, V_{BL} , are compared to experimental V_{50} results in Figure 8. The graph is plotted for both 12.7 mm and 20 mm FSP against UHMW-PE target thicknesses from 10 mm to 102 mm with respect to non-dimensional areal density (target areal density (AD_t) multiplied by the projectile presented area (A_p) divided by the projectile mass (m_p)). For thinner targets, the results are in excellent agreement with the experimental results. The results deviate slightly with increasing thickness and impact speed; however predictions are all within 20% of the experimental value and close to the experimental scatter range. A summary of the results showing the experimental V_{50} , experimental standard deviation ($\sigma_{V_{50}}$), numerical prediction (V_{BL}) and range, and percentage difference (% Diff) between the calculated and measured limit values is given in Table 5.

Figure 8: Experimental and numerical ballistic limit results plotted in terms of the non-dimensional areal density

Table 5. Comparison between experimental and predicted ballistic limit

Target Thickness (mm)	Threat	V_{50} (m/s)	σ_{V50} (m/s)	No. of sub-laminates in model	V_{BL} (m/s)	V_{BL} Range (m/s)	% Diff
9.1	12.7 mm FSP	506	26	7	450	450-500	-11.1
20	12.7 mm FSP	826	17	14	984	900-1000	19.1
25.2	12.7 mm FSP	1021	8	18	1096	1000-1100	7.3
35.1	12.7 mm FSP	1250	36	25	1387	1300-1400	11
50.4	12.7 mm FSP	1657	16	36	1800	1800-1900	8.6
10	20 mm FSP	394	43	5	394	394-440	0
20	20 mm FSP	620	20	10	626	605-684	1
36.2	20 mm FSP	901	10	18	1077	1020-1100	19.5
75.6	20 mm FSP	1528	105	36	1734	1660-1750	13.5
101.7	20 mm FSP	2002	92	51	2300	2300-2400	14.9

For numerical modelling of impacts just below the ballistic limit, one or more sub-laminates often delaminated fully from the target and were carried along with the projectile. Since the target was not perforated, this was considered a partial penetration result with a residual projectile velocity of 0 m/s. This was also observed in the ballistic experiments performed in Nguyen et al. [11], although the extent of delamination is not as significant. In the experiment, fibre bridging between sub-laminates provides significant resistance to complete delamination and therefore detachment of a subsection of the laminate. In the numerical

model, fibre bridging is not described, so the resistance of the sub-laminates to complete detachment is lower. This may explain the tendency of the numerical model to over-predict the ballistic limit, particularly for thicker targets with increasing numbers of sub-laminate interfaces, see Figure 9. Another reason for the over-prediction may be a lack of a thermal softening model, which describes the degradation of the mechanical properties of the composite. Important material properties such as the tensile strength and elastic modulus of UHMW-PE fibres, which are directly related to its ballistic performance [11], are degraded under thermal loads [31]. Under ballistic impact, thermal loads arise due to shock induced heating and plastic deformation of the material. Fibre and matrix melting have been observed around the penetration cavity of UHMW-PE composite targets [32] and post impact analysis of targets from test in [11] show targets impacted at higher velocities (or thicker targets) show greater levels of melting. The degradation in material hardening and failure strength is not account for using this current modelling approach.

Figure 9. A 102 mm thick target impacted by a 20 mm FSP below the ballistic limit. Numerical simulation at an impact velocity of 2200 m/s shown 750 μ s after impact. Full target (left) and cross-section of target (middle). The contours represent the status of the sub-laminate bonded interfaces with dark blue indicating failure. Post-test sample of target impacted at 1966 m/s showing complete delamination of sub-laminates (right).

Previous works [11] found that thick UHMW-PE composite targets (typically > 10 mm) are perforated in two successive stages by 12.7 mm and 20 mm FSPs; an initial stage characterised by localised deformation and failure, followed by a back face bulging stage. This is shown in the post-test target in Figure 9, where the back of the target forms a bulge and is detached from the front portion of the target, which experiences minimal global deformation. The transition between these two stages was identifiable because bulging of the back face caused the orthogonal fibres in contact with the projectile to be stretched and pulled in towards the centre. This plane separating the two modes of penetration was generally quite distinct and the proportion of the thickness undergoing the initial stage was measured and reported in [11] as the shear plugging thickness ratio. The same phenomenon was observed in the numerical model, and the ratio of the target penetrated under the initial stage was recorded and plotted with respect to the impact velocity in Figure 10.

Figure 10. Ratio of target thickness penetrated in shear plugging (t_s) to the total target thickness (t), with respect to the impact velocity.

For the numerical simulations, measurements were taken from one partial penetration and one complete perforation close to the ballistic limit for each target thickness and projectile combination. This is so a direct comparison can be made with the experimental results where the impact velocities were all very close to the V_{50} . Although there is a reasonable degree of scatter in both the experimental and numerical results, there is very good overall agreement between the two. Below impact velocities of around 500 m/s the UHMW-PE composite targets respond entirely through bulging. With increased impact velocity, the proportion of the target penetrated by shear plugging increases rapidly until it begins to plateau at about 75% of the total thickness above 1200 m/s.

3.2.1 Bulge development

High energy ballistic impact on UHMW-PE composite causes the back face to deform in a typically pyramidal shape for a cross-ply layup, as shown for example in Figure 11. The velocity at which the hinge of the bulge propagates was investigated for UHMW-PE composite strips and thin targets (up to four plies) by Chocron et al. [33]. The propagation velocity of the hinge was found to be the same as the transverse wave velocity, which is predicted using classical yarn theory [34]. For thicker targets the position and velocity of the hinge is more difficult to predict because of the added complexities of target bending resistance, a longer penetration process, and a projectile that is being significantly decelerated. Prediction of the bulge apex is important as it is an indication of the projectile position and velocity as it penetrates the target. Furthermore the bulge apex is an important parameter for many protection applications (i.e. back face deformation).

Figure 11. Numerical (left) and experimental (right) target response at 450 μ s after impact by 20 mm FSP at 888 m/s.

The hinge and apex position of a bulge from 10 mm, 20 mm and 36 mm thick Dyneema[®] HB26 targets impacted by a 20 mm FSP just below the ballistic limit were measured in-situ using a high speed camera in [11]. In these tests, the camera was positioned with a side view of the target, and a mirror was used to provide a perspective view of the back face on the same frame (as shown in Figure 11). A grid was drawn on the back of the target and the camera was calibrated for position measurements.

The geometric deformation of the bugle is well replicated in the numerical model, as shown in Figure 11. Figure 12 compares the hinge and apex positions with time following projectile impact determined from experiment and numerical simulation. The development of the bulge apex is well predicted by the model for the 10 mm and 20 mm thick panels, but less accurate for the 36 mm thick panel.

Figure 12: Bulge apex and hinge position with respect to time for 10 mm panel impacted at 365 m/s (a)(b), 20 mm panel impacted at 615 m/s (c)(d) and 36 mm panel impacted at 888 m/s (e)(f) by a 20 mm FSP.

As shown in the previous section, the ballistic limit prediction were more accurate for the 10 mm and 20 mm thick targets against the 20 mm FSP (within 1% error in V_{50} prediction) while the V_{50} for the 36 mm target was over-predicted by up to 20% (due to more sub-laminate interfaces and omission of thermal softening model as discussed above). The increased penetration resistance in the numerical model for the 36 mm thick target would be identified in the bulge growth by an under-prediction of the apex position, as seen in Figure 12(e). The hinge positions for the three cases investigated are in good agreement with experiment up to a hinge position of about 70 mm. For larger hinge displacement, numerical predictions deviate from the experiment because the propagation of the hinge is affected by target clamping (which is not modelled) as it extends close to the target edge. If the target (which measures 300 mm $\times$ 300 mm laterally) is impacted at the centre, the hinge can theoretically propagate up to 150 mm. However the presence of the vice clamps on the top

and bottom (as shown in Figure 11) of the target reduces this to about 100 mm for this configuration. Off-centre impact and resistance to rotation at the clamp edge can reduce further the zone where the hinge can propagate unimpeded.

4 Conclusion

An analysis methodology is proposed to model the behaviour of thick UHMW-PE composite panels under ballistic impact using the non-linear orthotropic model [7-9] in a commercial hydrocode. The approach provides a new material data set and improves on existing modelling methods by using sub-laminate discretisation of the target to better model delamination failure. It is introduced together with a newly implemented damage based erosion model which is more suitable for anisotropic materials.

The methodology was extensively validated against existing experimental ballistic impact data and results for UHMW-PE targets up to 102 mm thick. Depth of penetration into semi-infinite targets against 20 mm FSPs was predicted to within 3% of experimental values. Ballistic limit and residual velocity predictions were also in good agreement with experimental measurements, with the ballistic limit predictions within 20% for all conditions considered. The error may be attributed to the omission of both fibre bridging between the sub-laminate interfaces and possible thermal softening which is not prescribed by the numerical model. The two stages of penetration exhibited by thick UHMW-PE composite targets are well replicated with the model, which gives good predictions of the shear plugging thickness ratio. The bulge development of UHMW-PE composite under ballistic impact is accurately modelled in terms of the bulge hinge and apex position.

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