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A Discriminative Analysis of Approaches to Ranking Fuzzy Numbers in Fuzzy Decision Making

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Abstract

This paper presents a discriminative analysis of approaches to ranking fuzzy numbers in fuzzy decision making based on a comprehensive review of existing approaches. The consistency and effectiveness of the approaches to ranking fuzzy numbers are examined in terms of two objective measures developed, leading to a better understanding of the relative performance of individual approaches in ranking fuzzy numbers. Representative fuzzy numbers are selected for carrying out the comparative study of several typical approaches in ranking fuzzy numbers. Several interesting findings are identified which may be of practical significance to fuzzy decision making in real situations.

1. Introduction

Comparing and ranking fuzzy numbers for determining their overall rankings are an important part of fuzzy decision making [1, 2, 4, 13]. This is because these fuzzy numbers can be obtained in a fuzzy decision making situation to represent the overall utilities of decision alternatives, commonly referred to as fuzzy utilities [1, 2, 8, 13, 14, 18-24, 39, 68]. Most fuzzy decision making approaches developed in the context of multi-attribute utility theory [13, 18, 29] consist of the fuzzy utility aggregation process and the fuzzy utility comparison process. A comparison between fuzzy utilities (fuzzy numbers) thus is a comparison between decision alternatives [2, 18, 36, 49, 62, 68].

Numerous approaches have been developed for comparing and ranking fuzzy numbers. Freeling [27], Bortoland and Degani [5], Nakamura [46, 47], Lee and Li [38], Tseng *et al.* [53], Chen and Hwang [13], and Yeh and Deng [62] have conducted comprehensive reviews based on various classifications. In general all the ranking approaches produce sound ranking results for clear-cut problems [13, 68]. However, for problems where fuzzy numbers involved only differ slightly from each other, count-intuitive ranking outcomes may occur [13, 62, 68]. There is a lack of understanding of the relative performance of existing approaches, in particular with respect to the discrimination-ability of these approaches in

differentiating similar fuzzy numbers which is often critical in real decision situations [13, 62].

This paper presents a discriminative analysis of approaches to ranking fuzzy numbers for decision making in a fuzzy environment. The consistency and effectiveness of these approaches in ranking fuzzy numbers are examined in terms of two objective measures developed, leading to a better understanding of the performance of individual approaches. Representative fuzzy numbers are selected for carrying out the discriminative study of several typical approaches in ranking fuzzy numbers. Several interesting findings are identified which may be of practical significance to fuzzy decision making.

2. A review of ranking approaches

The significance of ranking fuzzy numbers for solving real world decision problems in a fuzzy environment [67] leads to the tremendous efforts being spent on the development of various ranking approaches [13, 62]. These approaches can be categorized into mathematical approaches and linguistic approaches [5, 13].

Linguistic approaches focus on the development and use of linguistic terms for describing the ranking outcome which is not ordinal [13, 50, 66, 68]. This study does not include linguistic approaches in the discriminative analysis study. Mathematical approaches consist of defining a ranking function for mapping a fuzzy number into a real one where a natural order exists [13], resulting in a single or multiple crisp index values [13, 68]. This study aims to examine the consistency and effectiveness of mathematical approaches to ranking fuzzy numbers.

Depending on the way that the ranking index is derived, mathematical approaches can further be divided into (a) independent ranking approaches, (b) reference-oriented ranking approaches, and (c) pairwise comparison based ranking approaches. Table 1 shows an overview of the classification of fuzzy ranking approaches.

The independent ranking approaches are the most common approach involving the development of a mapping function to associate a fuzzy number with a positive real number. The ranking of fuzzy numbers is based on comparing their corresponding real values. Approaches of Adamo [1], Buckley and Chanas [6],

Yager [58, 61], Chang [9], Murakami *et al.* [45], Lee and Li [38, 40], Campos and Munoz [7], and Liou and Wang [41] are in this category.

Table 1 A classification of fuzzy ranking approaches

Characteristics	Ranking Approaches
Independent Ranking	Adamo [1], Yager [59, 61], Chang [9], Murakami <i>et al.</i> [45], Lee and Li [38], Campos and Munoz [7], Liou and Wang [41].
Reference-oriented Ranking	Yager [60, 61], Jain [31, 32], Kerre [34], Chen [11], Kim and Park [35], Choobineh and Li [15], De Campos-Ibanez and Gonzalz-Munoz [15].
Pairwise Comparison Ranking	Bass and Kwakernaak [2], Baldwin and Guild [3], Watson <i>et al.</i> [57], Nakamura [46, 47], Kolodziejczyk [37], Tseng and Klein [52], Yuan [63], Dubois and Prade [23], Tsukamoto <i>et al.</i> [54], and Delgado <i>et al.</i> [17]
Linguistic Ranking	Zadeh [64, 65, 66], Freeling [27], Efstathiou and Tong [25], Tong and Bonissone [51]

With the use of reference-oriented ranking approaches, reference fuzzy sets are usually defined for establishing a common base to compare the fuzzy numbers in question during the ranking process. Each fuzzy number is evaluated based on its closeness to the reference sets [10, 18, 30, 33, 36, 55, 56, 62] via specifically developed mathematical functions, usually taking the form as follows $\mu_z(z_i) = \mu_i(y) * \mu_{A_i}(x_i)$ where Y is a reference set, $*$ denotes a given operator for defining the relationship between Y and A_i , and $Z_i = \{(z_i, \mu_{Z_i}(z_i)), z_i \in R\}$ is the resultant fuzzy set for each A_i to represent its relative ranking in relation to Y [62]. Approaches of Yager [60], Jain [31, 32], Kerre [34], Chen [11], Kim and Park [35], Choobineh and Li [15], Chen and Lu [12], de Campos-Ibanez and Gonzalez-Munoz [16], and Yeh and Deng [62] are developed based on this concept. Approaches in this group usually require considerable computation, particularly when continuous membership functions are present. Counter-intuitive ranking outcomes may occur under some circumstances [5, 13, 68].

Using pairwise comparison for determining the preference of alternatives is widely applied in fuzzy decision making. When all the pairwise comparison results are obtained, additional procedures can be developed to acquire a total order of alternatives. A typical application of this concept is the analytic hierarchy process [18, 48] based on the numerical scale of Miller [44]. Applying this concept to ranking fuzzy numbers has led to the development of several approaches, including those of Bass and Kwakernaak [2], Baldwin and Guild [3], Watson *et al.* [57], Tsukamoto *et al.* [54], Tseng and Klein [52], Nakamura [46, 47], Kolodziejczyk [37], Yuan [63], Dubois and Prade [23], and Delgado *et al.* [17]. These approaches usually involve in (a) constructing a fuzzy binary preference relation between two fuzzy

numbers based on their pairwise comparison and (b) determining the final ranking of all fuzzy numbers based on the fuzzy binary preference relations [13, 68].

When only two fuzzy numbers are present in the ranking process, the ranking relation between these two fuzzy numbers can be usually formulated as $A_1 > A_2 \Leftrightarrow P(A_1, A_2) > P(A_2, A_1)$, $A_1 = A_2 \Leftrightarrow P(A_1, A_2) = P(A_2, A_1)$, and $A_1 < A_2 \Leftrightarrow P(A_1, A_2) < P(A_2, A_1)$ where the fuzzy binary relation $P(A_i, A_j)$ representing the degree of preference of fuzzy numbers A_i to A_j can be developed in different manners. There are, however, difficulties when more than two fuzzy numbers are involved, as these fuzzy binary preference relations do not always abide the property of transitivity [13, 68]. In this situation, specific procedures are required for determining the overall ranking of fuzzy numbers during the ranking process.

Two ways are followed in the literature to address this problem for determining the total ranking order based on the available fuzzy binary preference relations. One is to use the *min* operator for aggregating the fuzzy binary relationships for determining the overall crisp rankings, represented by the approaches of Bass and Kwakernaak [2] and Baldwin and Guild [3]. The other approach to ranking fuzzy numbers based on available fuzzy binary preference relations is to introduce a certain transitivity property into the construction of the fuzzy preference relation $P(A_i, A_j)$ in order to eliminate the inconsistency which leads to a conflicting order relation such as $A_i > A_j$, $A_j > A_k$, and $A_k > A_i$. Approaches of Nakamura [46, 47], Kolodziejczyk [37], Tseng and Klein [52], Yuan [63], Dubois and Prade [23], Tsukamoto *et al.* [54], and Delgado *et al.* [17] are in this category.

These approaches are complicated in calculating the index for each fuzzy number. The computation involved is tremendous, in particular when various forms of fuzzy numbers are present. In some situations, these indices may also produce counter-intuitive ranking outcomes [13, 62].

Individual approaches in ranking fuzzy numbers have their own merits. The performances of these approaches are very different as they are developed based on different principles. To facilitate their applications in decision making for addressing real world problem, it is obviously desirable to conduct a discriminate analysis of these approaches for a better understanding of their performance, in particular their discrimination ability in differentiating similar fuzzy numbers.

3. Measures for discriminative analysis

The importance of understanding the performance of existing ranking approaches for fuzzy decision making is obvious. Two objective measures, the ranking consistency measure and the ranking effectiveness measure, are developed for carrying out the discriminative analysis. These two measures can be used to help explore the

performance of existing approaches which is useful for assisting the decision maker in selecting appropriate approaches for solving practical decision problems.

The ranking consistency measure aims to scrutinize the consistency of rankings of fuzzy numbers between each ranking approach and the intuitive ranking approach. It can be determined by comparing the ranking outcomes resulted from each ranking approach and the intuitive ranking approach, using Spearman's rank order correlation coefficient [3, 34, 43], given as

$$RC = 1 - \frac{\sum_{i=1}^n (g_i^d - g_i^N)^2}{n(n^2 - 1)} \quad (1)$$

The ranking effectiveness measure is designed to evaluate the discrimination-ability of individual approaches in differentiating fuzzy numbers. This measure is used to describe the degree to which fuzzy numbers are distinguished, based on the ranking values of fuzzy numbers. The measure is based on the perception that the decision maker would have more confidence in selecting a decision alternative when its corresponding ranking value is much larger than those of other alternatives [62].

For a set of n normalized ranking values P_i ($i = 1, 2, \dots, n$) listed in descending preference order, the ranking effectiveness measure ($RE_k \in [0, 1]$) can be calculated as

$$RE_k = \frac{\sum_{i=1}^k \frac{P_i - P_{i+1}}{\max(\frac{1}{k}, (P_i - P_{i+1}))}}{k}, k \in \{1, 2, \dots, n-1\}, \quad (2)$$

where k is an integer indicating the number of alternatives the decision maker wants to select out of n alternatives. RE_k is measured by the average ratio of the difference between each pair of adjacent ranking values P_i and P_{i+1} and the ideal number of fuzzy numbers to be differentiated. For example, only the first k ranking values requires the attention of the decision maker if he/she just wants to select the first k alternatives associated with fuzzy numbers. The larger the value of the ranking effectiveness measure is, the better the discrimination-ability of the ranking approach is, and the more confident the decision maker is in making the selection decisions.

4. A discriminative analysis

A discriminative analysis is carried out in this section to evaluate the relative performance of several typical approaches to ranking fuzzy numbers. A few approaches such as Buckley and Chanas's index [6], Mabuchi's index [43], and the multi-indices approaches presented by Dubois and Prade [23], Tsukamoto *et al.* [54], and Delgado *et al.* [17] are not included due to the practical limitation and complexity of the index themselves. Representative fuzzy numbers as shown in Figure 1 are selected from Bortoland and Degani [5], Nakamura [46,

47], Lee and Li [38], Tseng *et al.* [53], Chen and Hwang [13], and Yeh and Deng [62].

Figure 1 Example of representative cases

Adjustments on the indices are made to allow the fuzzy numbers ranked in an ascending order of the index value. Yager's index [59] and Kerre's index [34] are changed by subtracting their index values from 1 and the fuzzy numbers can be ranked in an ascending order of the revised index values. In Kolodziejczyk's indices $R_i(A_i, A_j)$ [37] is explained as the degree to which A_i is not preferred to A_j or is dominated by A_j . With the following changes, $R'_i(A_i, A_j)$ can be interpreted as the degree to which A_i is preferred to A_j or A_i dominate A_j . Chang's index [9] is multiplied by 2 to ensure that the comparability of the indices. Table 2 shows the computation results.

To conduct the discriminative analysis of these representative approaches in ranking fuzzy numbers, the ranking consistency and ranking effectiveness indices are calculated based on (1) and (2). Tables 3 and 4 present a comparison of these approaches in ranking fuzzy numbers with respect to these two measures.

Not a single approach dominates the other approaches with respect to the two objective performance measures, as shown in Tables 3 and 4. Their discrimination ability decreases when the fuzzy numbers involved are closer. Except the approach of Lee and Li [38], all other approaches have problems in separating similar fuzzy numbers. In general, the approaches involving the concept

of Hamming distance produce better results except that of Yager [60] and Kerre [34].

Table 3 Ranking consistency of typical approaches

Approaches	RC _A	RC _B	RC _C	RC _D	RC _E	RC _F	RC _G	RC _H	RC _I	RC _J
Bass/Kwakemaak	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.83
Baldwin-Guild	1.00	1.00	0.67	0.67	1.00	0.83	1.00	1.00	1.00	1.00
Watson	1.00	1.00	1.00	1.00	1.00	1.00	0.83	1.00	0.83	0.83
Yager [60]	1.00	0.67	1.00	0.67	0.83	1.00	0.67	0.83	0.67	0.67
Kerre	1.00	1.00	1.00	0.67	0.67	1.00	0.83	1.00	1.00	1.00
Nakamura	1.00	1.00	1.00	0.67	0.83	1.00	0.83	1.00	0.83	1.00
Kolodzijezyk R ₁	1.00	1.00	1.00	1.00	0.67	0.67	1.00	0.83	1.00	1.00
R ₂	1.00	1.00	1.00	1.00	0.83	0.67	1.00	0.83	1.00	1.00
R ₃	1.00	1.00	1.00	1.00	0.83	0.67	1.00	0.83	1.00	1.00
Tseng and Klein	1.00	1.00	1.00	0.67	0.67	1.00	0.83	1.00	1.00	1.00
Adamo $\alpha=0.50$	1.00	1.00	1.00	1.00	0.83	1.00	1.00	1.00	0.83	0.83
$\alpha=0.90$	1.00	1.00	1.00	1.00	1.00	1.00	0.63	1.00	0.83	0.83
Yager [58]	1.00	1.00	1.00	0.67	0.67	0.83	0.83	1.00	1.00	1.00
Murakami <i>et al.</i>	1.00	1.00	1.00	0.67	0.67	0.83	0.83	1.00	1.00	1.00
Yager [61]	1.00	1.00	1.00	0.67	0.83	1.00	0.83	1.00	1.00	1.00
Lee/ Li uniform	1.00	1.00	1.00	0.67	0.67	0.83	0.67	1.00	1.00	1.00
proportional	1.00	1.00	1.00	0.67	0.67	1.00	0.67	1.00	1.00	1.00
Chang	1.00	0.67	1.00	1.00	0.67	0.83	0.67	0.83	0.67	0.67
Jain $K = 1.0$	1.00	1.00	1.00	1.00	1.00	1.00	0.67	1.00	0.83	0.83
$k = 2.0$	1.00	1.00	1.00	1.00	0.83	1.00	0.67	1.00	0.83	0.83
$k = 0.5$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.83	0.83
Chen $k = 1.0$	1.00	1.00	1.00	1.00	1.00	1.00	0.83	1.00	1.00	1.00

Table 4 Ranking effectiveness of typical approaches

Approaches	RE _A	RE _B	RE _C	RE _D	RE _E	RE _F	RE _G	RE _H	RE _I	RE _J
Bass/Kwakemaak	1.00	1.00	1.00	0.18	0.24	0.66	0.00	0.04	0.00	0.00
Baldwin-Guild	0.92	0.74	0.08	0.06	0.08	0.00	0.02	0.08	0.18	0.10
Watson	1.00	0.00	1.00	0.18	0.24	0.66	0.00	0.04	0.00	0.00
Yager [60]	0.24	0.14	0.36	0.06	0.00	0.10	0.14	0.00	0.02	0.08
Kerre	0.60	0.90	0.90	0.12	0.02	0.12	0.00	0.04	0.10	0.30
Nakamura	1.00	1.00	1.00	0.56	0.00	1.00	0.00	1.00	1.00	1.00
Kolodzijezyk R ₁	1.00	1.00	1.00	0.12	0.04	0.24	0.00	0.08	0.24	0.56
R ₂	1.00	1.00	1.00	0.56	0.00	1.00	0.00	1.00	1.00	1.00
R ₃	1.00	1.00	1.00	0.12	0.00	0.56	0.00	0.12	0.24	0.56
Tseng and Klein	1.00	1.00	1.00	0.12	0.04	0.24	0.00	0.08	0.24	0.56
Adamo $\alpha=0.50$	0.70	0.20	0.60	0.10	0.00	0.20	0.26	0.02	0.00	0.00
$\alpha=0.90$	0.62	0.52	0.60	0.18	0.16	0.68	0.04	0.02	0.00	0.00
Yager [58]	0.60	0.60	0.60	0.14	0.06	0.00	0.00	0.02	0.06	0.14
Murakami <i>et al.</i>	0.30	0.30	0.30	0.06	0.02	0.00	0.00	0.02	0.04	0.08
Yager [61]	0.60	0.60	0.60	0.06	0.00	0.20	0.00	0.02	0.04	0.12
Lee/ Li uniform	0.60	0.60	0.60	0.14	0.06	0.00	0.16	0.02	0.06	0.16
proportional	0.60	0.60	0.60	0.06	0.02	0.20	0.12	0.02	0.04	0.16
Chang	0.32	0.68	0.36	0.20	0.04	0.00	0.40	0.00	0.48	0.26
Jain $K = 1.0$	0.80	0.30	0.46	0.08	0.06	0.36	0.10	0.02	0.00	0.00
$k = 2.0$	1.00	0.30	0.60	0.08	0.00	0.26	0.20	0.04	0.00	0.00
$k = 0.5$	0.56	0.24	0.32	0.06	0.06	0.32	0.04	0.02	0.00	0.00
Chen $k = 1.0$	0.84	0.54	0.50	0.02	0.06	0.36	0.00	0.02	0.06	0.26

5. Conclusion

This paper presents a comprehensive review of existing approaches to comparing and ranking fuzzy numbers. It provides a rational categorization of these approaches and conducts an investigation of the discrimination ability of

these approaches in differentiating similar fuzzy numbers. Not a single ranking approach appears to dominate others in terms of its ranking outcomes consistency and discrimination ability. Each approach has its own merit. An ideal ranking approach should be capable of (a) using as much information as possible provided by the fuzzy numbers involved, (b) providing intuitively consistent results to all people, (c) discriminating similar fuzzy numbers effectively, and (d) easy computation with understandable ranking outcomes.

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Table 2 Computation results of representative cases

Approaches	(A)		(B)		(C)		(D)		(E)		(F)		(G)		(H)		(I)		(J)	
	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂	A ₁	A ₂
Bass-Kwakemaak	1.00	0.00	1.00	0.50	1.00	0.50	1.00	0.91	1.00	0.88	1.00	0.67	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00
Baldwin-Guild	0.46	0.00	0.56	0.19	0.44	0.48	0.36	0.39	0.40	0.36	0.38	0.38	0.29	0.28	0.33	0.29	0.38	0.29	0.38	0.33
Watson	1.00	0.00	0.50	0.00	0.50	0.00	0.09	0.00	0.12	0.00	0.33	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
Yager [60]	0.59	0.47	0.59	0.66	0.74	0.56	0.67	0.69	0.64	0.64	0.67	0.62	0.59	0.66	0.56	0.56	0.64	0.65	0.64	0.68
Kerre	1.00	0.70	0.99	0.54	1.00	0.55	0.89	0.95	0.95	0.96	0.93	0.87	0.90	0.90	1.00	0.98	1.00	0.95	1.00	0.85
Nakamura	1.00	0.00	0.98	0.02	1.00	0.00	0.36	0.64	0.50	0.50	0.80	0.20	0.50	0.50	1.00	0.00	1.00	0.00	1.00	0.00
Kolodzizezyk R ₁	1.00	0.00	0.87	0.13	0.87	0.13	0.47	0.53	0.49	0.51	0.56	0.44	0.50	0.50	0.52	0.48	0.56	0.44	0.64	0.36
R ₂	1.00	0.00	0.98	0.02	1.00	0.00	0.36	0.64	0.50	0.50	0.80	0.20	0.50	0.50	1.00	0.00	1.00	0.00	1.00	0.00
R ₃	1.00	0.00	0.90	0.10	0.90	0.10	0.47	0.53	0.50	0.50	0.64	0.36	0.50	0.50	0.53	0.47	0.56	0.44	0.64	0.36
Tseng and Klein	1.00	0.00	0.87	0.13	0.87	0.13	0.47	0.53	0.49	0.51	0.56	0.44	0.50	0.50	0.52	0.48	0.56	0.44	0.64	0.36
Adamo $\alpha=0.50$	0.60	0.25	0.85	0.75	0.85	0.55	0.75	0.70	0.65	0.65	0.70	0.60	0.60	0.47	0.51	0.50	0.70	0.70	0.70	0.70
$\alpha=0.90$	0.52	0.21	0.81	0.55	0.73	0.43	0.55	0.46	0.53	0.45	0.70	0.36	0.52	0.54	0.43	0.42	0.62	0.62	0.62	0.62
Yager [58]	0.50	0.20	0.80	0.50	0.70	0.40	0.50	0.57	0.50	0.53	0.50	0.50	0.50	0.50	0.41	0.40	0.60	0.57	0.60	0.53
Murakami <i>et al.</i>	0.42	0.27	0.57	0.42	0.52	0.37	0.42	0.45	0.42	0.43	0.42	0.42	0.42	0.42	0.37	0.36	0.47	0.45	0.47	0.43
Yager [61]	0.50	0.20	0.80	0.50	0.70	0.40	0.50	0.53	0.50	0.50	0.55	0.45	0.50	0.50	0.50	0.49	0.51	0.49	0.53	0.47
Lee and Li uniform	0.50	0.20	0.80	0.50	0.70	0.40	0.50	0.57	0.50	0.53	0.50	0.50	0.08	0.16	0.41	0.40	0.60	0.57	0.60	0.53
proportional	0.50	0.20	0.80	0.50	0.70	0.40	0.50	0.53	0.10	0.11	0.55	0.45	0.06	0.12	0.41	0.40	0.60	0.58	0.60	0.52
Chang	0.20	0.04	0.16	0.50	0.42	0.24	0.50	0.40	0.30	0.32	0.30	0.30	0.20	0.40	0.16	0.16	0.24	0.48	0.24	0.37
Jain $K = 1.0$	0.78	0.38	0.82	0.67	0.77	0.54	0.67	0.63	0.67	0.64	0.78	0.60	0.64	0.69	0.75	0.74	0.80	0.80	0.80	0.80
$k = 2.0$	0.66	0.16	0.69	0.54	0.65	0.35	0.54	0.50	0.51	0.51	0.60	0.47	0.46	0.56	0.63	0.61	0.69	0.69	0.69	0.69
$k = 0.5$	0.87	0.59	0.90	0.78	0.86	0.70	0.78	0.75	0.79	0.76	0.88	0.72	0.78	0.80	0.85	0.84	0.88	0.88	0.88	0.88
Chen $k = 1.0$	0.63	0.21	0.77	0.50	0.63	0.38	0.50	0.49	0.45	0.42	0.59	0.41	0.50	0.50	0.51	0.50	0.57	0.54	0.57	0.44