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In his 1942 paper on the sound insulation of single leaf walls, Cremer [(1942). *Akust. Z.* **7**, 81–104] made a number of approximations in order to show the general trend of sound insulation above the critical frequency. Cremer realized that these approximations limited the application of his theory to frequencies greater than twice the critical frequency. This paper removes most of Cremer's approximations so that the revised theory can be used down to the critical frequency. The revised theory is used as a correction to the diffuse field limp panel mass law below the critical frequency by setting the nonexistent coincidence angle to 90°. The diffuse field limp panel mass law for a finite size wall is derived without recourse to a limiting angle by following the average diffuse field single sided radiation efficiency approach. The shear wave correction derived by Heckl and Donner [(1985). *Rundfunktech Mitt.* **29**, 287–291] is applied to the revised theory in order to cover the case of thicker walls. The revised theory predicts the general trend of the experimental data, although the agreement is usually worse at low frequencies and depends on the value of damping loss factor used in the region of and above the critical frequency.

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I. INTRODUCTION

Cremer's (1942) theory for the sound insulation of thin walls above the critical frequency is still in widespread use today. A number of other authors have also derived methods for predicting the sound insulation of thin walls. Important contributions include those of Crocker and Price (1969), Sewell (1970), Leppington *et al.* (1987), and Villot *et al.* (2001).

A critical examination of Cremer's (1942) theory shows that Cremer (1942) made a number of approximations which limit the application of his theory to frequencies which are greater than twice the critical frequency. Cremer (1942) stated this limitation in his paper. The last of Cremer's (1942) approximations was to assume that the single side radiation efficiency of a panel above the critical frequency was unity. This enables Cremer's (1942) theory to be used at and just below the critical frequency, although most of the approximations are obviously incorrect in this frequency region. It is necessary to use Cremer's (1942) approximate theory at and just below the critical frequency because Sewell's (1970) approximate correction factor for just below the critical frequency predicts infinite transmission at the critical frequency.

In this paper, most of Cremer's (1942) approximations are removed and the single sided radiation efficiency of an infinite panel is replaced with that for a finite panel. This produces a theory which is more exact and which can be used down to the critical frequency.

Below the critical frequency, the need to use a limiting angle is avoided by following the average diffuse field single

sided radiation efficiency approach. Instead of using Sewell's (1970) approximate correction factor for just below the critical frequency, the more exact version of Cremer's (1942) theory for above the critical frequency, which is developed in this paper, is added to the theory below the critical frequency. This is made possible by setting the coincidence angle, which does not exist below the critical frequency, equal to 90°. A major advantage of this approach is that there is only a very slight discontinuity at the critical frequency.

In order to account for the transition from bending waves to shear waves which occurs in thick panels at high frequencies, this paper follows Ljunggren's (1991) approach and uses the shear wave correction factor developed by Heckl and Donner (1985).

II. ABOVE THE CRITICAL FREQUENCY

The sound transmission coefficient $\tau(\theta)$ of a wall is the ratio of the sound energy transmitted by the wall to the sound energy incident upon the wall. For an infinite, isotropic, uniform thickness plane wall the sound transmission coefficient of a plane wave depends on the angle θ between the direction of propagation of the incident plane wave and the normal to the plane of the wall. To evaluate the diffuse field sound transmission coefficient τ_d it is necessary to average the plane wave sound transmission coefficient $\tau(\theta)$ with appropriate weighting across all angles of incidence [Cremer (1942), Eq. (4.9)],

$$\tau_d = 2 \int_0^{\pi/2} \tau(\theta) \cos \theta \sin \theta d\theta. \quad (1)$$

The $\cos \theta$ term is the cross-sectional area of the plane sound wave that is incident on a unit area of the wall at an angle of incidence of θ to the normal to the wall. The $\sin \theta$ term is due to the fact that the annulus of solid angle between θ and

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$\theta + \Delta\theta$ is $2\pi \sin \theta \Delta\theta$. The 2 term is a normalization factor which arises from the fact that τ_d must be 1 when $\tau(\theta)$ is 1 for all values of θ . Equation (1) can be rewritten in a number of forms. Use will be made of the following form [Cremer (1942), Eq. (4.9)]:

$$\tau_d = \int_0^1 \tau(\theta) d(\cos^2 \theta). \quad (2)$$

For a thin plane wall with the properties described above Cremer's (1942) Eq. (4.8) is

$$\tau(\theta) = \frac{1}{\left| 1 + \frac{Z(\theta) \cos \theta}{2\rho_0 c} \right|^2}, \quad (3)$$

where $Z(\theta)$ is the bending wave impedance of the wall and $\rho_0 c$ is the characteristic impedance of air, being the product of the ambient density ρ_0 and the speed of sound in air c . For an infinite panel, the single sided radiation efficiency $\sigma(\theta)$ is

$$\sigma(\theta) = \frac{1}{\cos^2 \theta}. \quad (4)$$

Thus Eqs. (1) and (3) can be rewritten as

$$\tau_d = 2 \int_0^{\pi/2} \frac{\tau(\theta)}{\sigma(\theta)} \sin \theta d\theta \quad (5)$$

and

$$\tau(\theta) = \frac{1}{\left| 1 + \frac{Z(\theta)}{2\rho_0 c \sigma(\theta)} \right|^2}. \quad (6)$$

Cremer's (1942) Eq. (9.2) can be written as

$$Z(\theta) = m\omega[j(1 - r^2 \sin^4 \theta) + \eta r^2 \sin^4 \theta], \quad (7)$$

where

$$r = \frac{\omega}{\omega_c} = \frac{f}{f_c}. \quad (8)$$

$f_c(\omega_c)$ is the (angular) critical frequency of the panel, $f(\omega)$ is the (angular) frequency of the sound, and m is the mass per unit area of the single leaf panel. η is the total damping loss factor of the single leaf panel which is equal to the sum of the internal damping loss factor of the panel η_{int} , the damp-

ing loss factor η_{edge} due to the transmission of vibrational energy from the panel to its surrounding elements at its edges and twice (to take account of both sides of the panel) its single sided radiation loss factor η_{rad} . Thus

$$\eta = \eta_{\text{int}} + \eta_{\text{edge}} + 2\eta_{\text{rad}}. \quad (9)$$

The single sided radiation loss factor is related to the single sided radiation efficiency by

$$\eta_{\text{rad}} = \frac{\sigma(\theta) \rho_0 c}{m\omega}. \quad (10)$$

Annex C of ISO 15712-1:2005(E) (ISO, 2005) gives guidance on the calculation of the total damping loss factor. For the laboratory situation of a panel with a surface density m of less than 800 kg/m^2 , it says that

$$\eta = \eta_{\text{int}} + \eta_{\text{edge}} = \eta_{\text{int}} + \frac{m}{485\sqrt{f}}, \quad (11)$$

where η_{int} is the internal loss factor of the panel material and can normally be taken as 0.01. Equation (11) ignores the radiation loss factor because it is usually insignificant.

Inserting Eq. (7) into Eq. (6) produces Cremer's (1942) Eq. (9.3) for the sound transmission coefficient $\tau(\theta)$ of a single leaf panel as a function of angle of incidence θ .

$$\tau(\theta) = \frac{1}{\left(1 + \frac{a\eta r^2 \sin^4 \theta}{\sigma(\theta)} \right)^2 + \left(\frac{a}{\sigma(\theta)} \right)^2 (1 - r^2 \sin^4 \theta)^2}, \quad (12)$$

where

$$a = \frac{\omega m}{2\rho_0 c} = \frac{\pi f m}{\rho_0 c}. \quad (13)$$

For a given frequency which is greater than or equal to the critical frequency, the maximum value of Eq. (12) occurs at the coincidence angle θ_c where

$$\sin^2 \theta_c = \frac{1}{r} = \frac{\omega_c}{\omega} = \frac{f_c}{f}. \quad (14)$$

For values of θ which are close to θ_c , Eq. (12) can be approximated by setting most of the values of θ which occur in Eq. (12) equal to θ_c . This gives

$$\tau(\theta) \approx \frac{1}{\left(1 + \frac{a\eta r^2 \sin^4 \theta_c}{\sigma(\theta_c)} \right)^2 + \left(\frac{a}{\sigma(\theta_c)} \right)^2 (1 - r \sin^2 \theta)^2 (1 + r \sin^2 \theta_c)^2} = \frac{1}{\left(1 + \frac{a\eta}{\sigma(\theta_c)} \right)^2 + \left(\frac{2ar}{\sigma(\theta_c)} \right)^2 \left(\frac{1}{r} - \sin^2 \theta \right)^2}. \quad (15)$$

Put

$$x = \cos^2 \theta. \quad (16)$$

Then

$$\sin^2 \theta = 1 - x, \quad (17)$$

and

$$\tau(\theta) = \frac{1}{\left(1 + \frac{a\eta}{\sigma(\theta_c)}\right)^2 + \left(\frac{2ar}{\sigma(\theta_c)}\right)^2 \left(x + \frac{1}{r} - 1\right)^2}. \quad (18)$$

Putting Eq. (18) into Eq. (2) gives the diffuse field sound transmission coefficient as

$$\tau_d = \int_0^1 \frac{dx}{\left(1 + \frac{a\eta}{\sigma(\theta_c)}\right)^2 + \left(\frac{2ar}{\sigma(\theta_c)}\right)^2 \left(x + \frac{1}{r} - 1\right)^2}. \quad (19)$$

Put

$$y = x + \frac{1}{r} - 1, \quad (20)$$

then

$$dy = dx, \quad (21)$$

and

$$\tau_d = \int_{1/r-1}^{1/r} \frac{dy}{\left(1 + \frac{a\eta}{\sigma(\theta_c)}\right)^2 + \left(\frac{2ar}{\sigma(\theta_c)}\right)^2 y^2}. \quad (22)$$

This paper departs from [Cremer \(1942\)](#) by not approximating by extending the range of integration to cover from $-\infty$ to ∞ . Using integral number 2.124.1 on page 60 of [Gradshteyn and Ryzhik \(1965\)](#),

$$\tau_d = \left[\frac{1}{\frac{2ar}{\sigma(\theta_c)} \left(1 + \frac{a\eta}{\sigma(\theta_c)}\right)} \arctan \left(\frac{2arx}{\sigma(\theta_c) + a\eta} \right) \right]_{1/r-1}^{1/r}. \quad (23)$$

Thus

$$\tau_d = \frac{\sigma^2(\theta_c)}{2ar(\sigma(\theta_c) + a\eta)} \left\{ \arctan \left[\frac{2a}{\sigma(\theta_c) + a\eta} \right] - \arctan \left[\frac{2a(1-r)}{\sigma(\theta_c) + a\eta} \right] \right\}, \quad (24)$$

or

$$\tau_d = \frac{\left(\frac{\sigma(\theta_c)}{a}\right)^2}{2r\left(\frac{\sigma(\theta_c)}{a} + \eta\right)} \left\{ \arctan \left[\frac{2}{\frac{\sigma(\theta_c)}{a} + \eta} \right] - \arctan \left[\frac{2(1-r)}{\frac{\sigma(\theta_c)}{a} + \eta} \right] \right\}. \quad (25)$$

This paper will use Eq. (24) or Eq. (25). However, for comparison [Cremer's \(1942\)](#) approximations will now be derived. The integrand in Eq. (22) is a maximum when $y=0$. If $a\eta/\sigma(\theta) \gg 1$, which is usually the case, the integrand is half its maximum value when $|y| = \eta/2r$. Since η is usually very much less than 1 and r is greater than or equal to 1 if the frequency is greater than or equal to the critical frequency, the values of y where the integrand is significantly different from zero usually lie well inside the integral limits from

$1/r-1$ to $1/r$. Because of this [Cremer \(1942\)](#) approximated the integral in Eq. (22) by extending the limits of integration from $-\infty$ to $+\infty$.

$$\tau_d \approx \int_{-\infty}^{\infty} \frac{dy}{\left(1 + \frac{a\eta}{\sigma(\theta_c)}\right)^2 + \left(\frac{2ar}{\sigma(\theta_c)}\right)^2 y^2}. \quad (26)$$

With this approximation Eqs. (24) and (25) become

$$\tau_d \approx \frac{\pi \sigma^2(\theta_c)}{2ar(\sigma(\theta_c) + a\eta)}, \quad (27)$$

and

$$\tau_d \approx \frac{\pi \left(\frac{\sigma(\theta_c)}{a}\right)^2}{2r \left(\frac{\sigma(\theta_c)}{a} + \eta\right)}. \quad (28)$$

Equation (27) is the same as [Cremer's \(1942\)](#) Eq. (9.6). [Cremer \(1942\)](#) also assumed the usual case of $a\eta/\sigma \gg 1$, which gives

$$\tau_d \approx \frac{1}{a^2} \frac{\pi}{2\eta} \frac{\sigma^2(\theta_c)}{r}. \quad (29)$$

Because [Cremer \(1942\)](#) assumed an infinite panel above the critical frequency, he also used the radiation efficiency of free bending waves above the critical frequency for an infinite panel. This is because the wavelength of the forced waves at coincidence is equal to the free bending wavelength.

$$\sigma(\theta_c) = \frac{1}{\sqrt{1 - \frac{1}{r}}} = \frac{1}{\sqrt{1 - \frac{\omega_c}{\omega}}} = \frac{1}{\sqrt{1 - \frac{f_c}{f}}} = \frac{1}{\cos \theta_c}. \quad (30)$$

Substituting Eq. (30) into Eq. (29) gives [Cremer's \(1942\)](#) Eq. (9.8).

$$\tau_d \approx \frac{1}{a^2} \frac{\pi}{2\eta} \frac{1}{r-1}. \quad (31)$$

When $r > 2$, [Cremer \(1942\)](#) approximated Eq. (31) as

$$\tau_d \approx \frac{1}{a^2} \frac{\pi}{2\eta} \frac{1}{r}. \quad (32)$$

This last approximation of [Cremer \(1942\)](#) is equivalent to assuming that the radiation efficiency of a panel above its critical frequency is equal to unity. Equation (32) is a version of [Cremer's \(1942\)](#) Eq. (9.10).

At the critical frequency, $r=1$ and the lower limit of integration in Eq. (22) is zero. Because of the symmetrical nature of the integrand about zero, [Cremer's \(1942\)](#) extension of the limits of integration produces a value of the transmission coefficient at the critical frequency which is more than twice as large as the value before the extension of the limits of integration. Thus in this paper, Eq. (24) or Eq. (25)

will be used instead of [Cremer's \(1942\)](#) further approximations which are given by Eqs. (27)–(29), Eq. (31), or Eq. (32).

Because Eq. (30) gives an infinite value for the radiation efficiency at the critical frequency, this paper uses [Davy's \(2004\)](#) theory to calculate the radiation efficiency. The equations given here are an updated version of those in [Davy \(2004\)](#). First the cosine of the coincidence angle is calculated. Since Eq. (24) or Eq. (25) is going to be used as a correction term below the critical frequency, the cosine of the coincidence angle is set to zero for frequencies below the critical frequency.

$$g = \begin{cases} \cos \theta_c = \sqrt{1 - \frac{\omega_c}{\omega}} = \sqrt{1 - \frac{f_c}{f}} & \text{if } \omega \geq \omega_c \\ 0 & \text{if } \omega < \omega_c. \end{cases} \quad (33)$$

Then three empirical constants are defined. These empirical constants are chosen to make Eq. (39) agree as closely as possible with numerical calculations by [Sato \(1973\)](#). Sato's numerical results appear as Fig. 1.3.2, Table B.1, and Fig. B.2 in [Rindel \(1975\)](#). The three empirical constants are $n=2$, $w=1.3$, and $\beta=0.124$.

The length of the side of an equivalent square panel is defined.

$$2a = \frac{4S}{U}, \quad (34)$$

where S is the area of the panel and U is the perimeter of the panel.

Some intermediate values are calculated using the wave number $k = \omega/c = 2\pi f/c$.

$$p = \begin{cases} w \sqrt{\frac{\pi}{2ka}} & \text{if } w \sqrt{\frac{\pi}{2ka}} \leq 1 \\ 1 & \text{if } w \sqrt{\frac{\pi}{2ka}} > 1. \end{cases} \quad (35)$$

$$h = \frac{1}{\frac{2}{3} \sqrt{\frac{2ka}{\pi}} - \beta}. \quad (36)$$

$$\alpha = \frac{h}{p} - 1. \quad (37)$$

$$q = \frac{2\pi}{k^2 S}. \quad (38)$$

Finally the radiation efficiency is calculated.

$$\sigma(\theta_c) = \begin{cases} \frac{1}{\sqrt[n]{g^n + q^n}} & \text{if } 1 \geq g \geq p \\ \frac{1}{\sqrt[n]{(h - \alpha g)^n + q^n}} & \text{if } p > g \geq 0. \end{cases} \quad (39)$$

III. BELOW THE CRITICAL FREQUENCY

The sound transmission coefficient below the critical frequency is calculated using the average diffuse field single sided radiation efficiency approach. Bending stiffness is ignored by setting r equal to zero in Eq. (7). Bending stiffness will be included later on by adding in Eq. (24) or Eq. (25). Equation (7) becomes

$$Z(\theta) = jm\omega. \quad (40)$$

Substituting Eq. (40) into Eq. (6), and assuming that the second term in the modulus brackets is much greater than 1, gives

$$\tau(\theta) = \left(\frac{2\rho_0 c \sigma(\theta)}{m\omega} \right)^2 = \left(\frac{\sigma(\theta)}{a} \right)^2. \quad (41)$$

Substituting Eq. (41) into Eq. (5) gives

$$\tau_d = \frac{2}{a^2} \int_0^{\pi/2} \sigma(\theta) \sin \theta d\theta = \frac{2\langle\sigma\rangle}{a^2}, \quad (42)$$

where

$$\langle\sigma\rangle = \int_0^{\pi/2} \sigma(\theta) \sin \theta d\theta. \quad (43)$$

Substituting Eq. (39) in Eq. (43) gives

$$\langle\sigma\rangle = \ln \left(\frac{1 + \sqrt{1 + q^2}}{p + \sqrt{p^2 + q^2}} \right) + \frac{1}{\alpha} \ln \left(\frac{h + \sqrt{h^2 + q^2}}{p + \sqrt{p^2 + q^2}} \right). \quad (44)$$

An earlier version of this equation is given in [Davy \(2004\)](#).

To include the effects of bending stiffness, the sound transmission coefficient below the critical frequency is calculated as the sum of Eq. (42) and Eq. (24) or Eq. (25). The use of Eq. (24) or Eq. (25) as the correction for bending stiffness below the critical frequency is a new approach introduced by this paper. Above the critical frequency, only Eq. (24) or Eq. (25) is used. Equation (42) can be included immediately above the critical frequency, but it causes problems if used a long way above the critical frequency.

Because of the rapid variation in the sound transmission coefficient with frequency near the critical frequency, if the measurement frequency band includes the critical frequency, the sound transmission coefficient is averaged over the measurement frequency band. In this paper for comparison with measurements in the third octave band which included the critical frequency, the theoretical values were averaged over the three frequencies which were $2^{-1/9}$, 1, and $2^{1/9}$ times the center frequency of the third octave band.

IV. SHEAR WAVE CORRECTION

To account for the transition from bending waves to shear waves, Heckl and Donner's correction ([Ljunggren, 1991](#)) is applied to Eqs. (24) and (25). According to [Ljunggren \(1991\)](#),

“the Mindlin plate theory was applied to the case of an infinite wall by Heckl and Donner in the same way as the simple bending wave theory was applied in Cremer's paper from 1942.

In these equations, k_M is the wave number at coincidence, that is, the “corrected” wave number of the free bending wave [see for example, [Cremer et al. \(1988\)](#), p. 109], k_T is the wave number of a hypothetical, corrected shear wave,

$$k_T^2 = \frac{\omega^2 \rho}{G^*}, \quad (45)$$

where G^* is the shear modulus, modified to account for shear distribution, and

$$k_S^2 = k_T^2 + k_L^2. \quad (46)$$

The wave number k_B is ... the wave number of the free bending wave according to the thin plate theory. ρ is the density of the plate material.”

k_L is the wave number of the quasilongitudinal wave. G^* is calculated from the following expressions [[Magrab \(1979\)](#), p. 281]:

$$G^* = \frac{G}{\chi}, \quad (47)$$

$$\chi = \left(\frac{1 + \nu}{0.87 + 1.12\nu} \right)^2, \quad (48)$$

where G is the shear modulus and ν is Poisson’s ratio.

$$A = \left[1 + \frac{\delta^2}{12} \left(\frac{k_M^2 k_T^2}{k_L^2} - k_T^2 \right) \right]^2, \quad (49)$$

$$B = 1 - \frac{(k_T \delta)^2}{12} + \frac{k_M^2 k_S^2}{k_B^4}, \quad (50)$$

$$C = \sqrt{1 - \frac{(k_T \delta)^2}{12} + \frac{k_S^2}{4k_B^4}}, \quad (51)$$

where δ is the thickness of the panel. Equations (24) and (25) are multiplied by the correction factor

$$\frac{A}{BC}. \quad (52)$$

k_M^2 is the positive solution of the following quadratic equation ([Cremer et al., 1988](#)):

$$\frac{E}{(1 - \nu^2)\rho} \frac{\delta^2}{12} k_M^4 - \omega^2 \left(1 + \frac{2\chi}{1 - \nu} \right) \frac{\delta^2}{12} k_M^2 - \omega^2 = 0, \quad (53)$$

where E is Young’s modulus.

The uncorrected shear modulus is

$$G = \frac{E}{2(1 + \nu)}. \quad (54)$$

k_B^4 is given by

$$k_B^4 = \frac{12\omega^2 \rho (1 - \nu^2)}{E \delta^2}. \quad (55)$$

k_L^2 is given by

$$k_L^2 = \frac{\omega^2 \rho (1 - \nu^2)}{E}. \quad (56)$$

FIG. 1. The sound insulation of 200 mm thick concrete. The experimental results are from [Ljunggren \(1991\)](#). The theory1 curve is calculated using the *in-situ* measured total damping loss factor. The theory2 curve is calculated using the total damping loss factor given by Eq. (11).

m , ρ , and δ are related by $m = \rho \delta$. The angular critical frequency is given by

$$\omega_c = \frac{c^2}{\delta} \sqrt{\frac{12\rho(1 - \nu^2)}{E}}. \quad (57)$$

This equation can be inverted to calculate Young’s modulus from the angular critical frequency. If Poisson’s ratio is not known, assume that it is 0.3. Most materials have Poisson’s ratios that are close to 0.3.

V. COMPARISON WITH EXPERIMENT

At and above the critical frequency, the theoretical values were calculated using Eq. (24) or Eq. (25) multiplied by Eq. (52). Below the critical frequency Eq. (44) was also added to obtain the final theoretical result. Figure 1 compares the experimental sound insulation of a 200 mm thick concrete wall ([Ljunggren, 1991](#)) with the theory described in this paper. There are two theoretical curves. The theory1 curve is calculated using the *in-situ* measured total damping loss factor, while the theory2 curve is calculated using the total damping loss factor given by Eq. (11). The theoretical calculations assume that the density is 2300 kg/m^3 , Young’s modulus is $1.36 \times 10^{10} \text{ Pa}$, and Poisson’s ratio is 0.3. The specimen size is $4.08 \times 3.08 \text{ m}^2$. Below 180 Hz, the experimental measurements are substantially lower than the theoretical results. It should be noted that the experimental uncertainties are larger in this frequency region and that [Ljunggren \(1991\)](#) did not attempt to compare theory and experiment below 315 Hz. Above 180 Hz, not surprisingly, the theory1 results, which are calculated using the measured total damping loss factor, are in better agreement with the experimental results than the theory2 results which use predicted total damping loss factors. Nevertheless, the theory2 results still give a good estimate of the general trend of the experimental results.

Figure 2 compares the theory with the measured sound insulation of a single layer of 13 mm gypsum plaster board. The experimental results were measured by the National Research Council of Canada (NRCC) ([Northwood, 1968](#); [Dupree, 1981](#)). One of the measurements is on a wall with no studs, while the other two measurements have wooden studs

FIG. 2. The sound insulation of 13 mm gypsum plasterboard without studs and with studs spaced at 400 and 600 mm centers. The experimental results were measured by the NRCC (Northwood, 1968; Dupree, 1981).

spaced at 400 and 600 mm, respectively. The no stud results are the average of three separate measurements. The theoretical calculations assume that the density is 770 kg/m^3 , Young's modulus is $1.85 \times 10^9 \text{ Pa}$, Poisson's ratio is 0.3, and the total damping loss factor is 0.04. This value of the total damping loss factor is chosen to give the best agreement between theory and experiment. It is slightly greater than the usually accepted range for gypsum plaster board from 0.01 to 0.03. The specimen size used in the calculations is $3.05 \times 2.44 \text{ m}^2$. The experimental results show that the wooden studs do not make any significant difference, while the theory slightly but significantly overestimates the experimental results in the lower frequency range.

Figure 3 compares theory with three measurements of the sound insulation of 6 mm monolithic glass by Monsanto, Pilkington, and the NRCC (Quirt, 1981, 1982). The theoretical calculations assume that the density is 2500 kg/m^3 , Young's modulus is $6.5 \times 10^{10} \text{ Pa}$, Poisson's ratio is 0.22, and the total damping loss factor is 0.05. Again, this value of the total damping loss factor is chosen to give the best agreement between theory and experiment. A specimen size measuring $1.8 \times 1.2 \text{ m}^2$ is assumed for the calculations. Although the total damping loss factor seems high for glass, it should be noted that Cremer (1942) assumed a damping loss factor of 0.1. The author's experience is that it is often nec-

FIG. 3. The sound insulation of 6 mm monolithic glass. The experimental results were measured by Monsanto, Pilkington, and the NRCC (Quirt, 1981, 1982).

FIG. 4. The difference in decibels of the sound reduction index R of 6 mm monolithic glass calculated using Eq. (29), Eq. (31), or Eq. (32) compared to that calculated using Eq. (25).

essary to use higher damping loss factors than would be measured directly in order to make theoretical predictions and experimental measurements of diffuse field sound insulation and the directivity of sound insulation agree. When the scatter of the experimental results is taken into account the agreement between theory and experiment is good.

Figure 4 shows the difference in decibels of the sound reduction index R calculated using Eq. (29), Eq. (31), or Eq. (32) compared to that calculated using Eq. (25). These calculations are for the specimen shown in Fig. 3 since the results depend on the properties of the specimen. At the critical frequency, the result calculated using Eq. (29) is 3 dB less than the result calculated using Eq. (25). The result calculated using Eq. (31) is $-\infty \text{ dB}$ at the critical frequency.

VI. CONCLUSIONS

This paper develops a more exact version of Cremer's (1942) theory of sound insulation which allows Cremer's (1942) theory to be used down to the critical frequency and as a correction below the critical frequency. The approach adopted in this paper avoids large discontinuities in the region of the critical frequency. This more exact version of the theory agrees reasonably well with the experimental data presented, although there are some systematic differences. The use of the single sided forced radiation efficiency for a finite sized panel below the critical frequency avoids the need to use a limiting angle of integration. Applying Heckl and Donner's correction (Ljunggren, 1991) for the transition from bending to shear waves works well for the case of a thicker 200 mm concrete wall.

Cremer, L. (1942). "Theorie der schalldämmung wände bei schrägem einfall," Akust. Z. 7, 81–104. Most of this article has been republished with an English language summary in: Northwood, T. D. (1977). "Theory of the sound attenuation of thin walls with oblique incidence," in *Architectural Acoustics*, Benchmark Papers in Acoustics Vol. 10, edited by T. D. Northwood (Dowden, Hutchinson and Ross, Stroudsburg, PA), pp. 367–399.

Cremer, L., Heckl, M., and Ungar, E. E. (1988). *Structure-Borne Sound: Structural Vibrations and Sound Radiation at Audio Frequencies*, 2nd ed. (Springer-Verlag, Berlin).

Crocker, M. J., and Price, A. J. (1969). "Sound transmission using statistical energy analysis," J. Sound Vib. 9, 469–486.

Davy, J. L. (2004). "The radiation efficiency of finite size flat panels," in *Acoustics 2004, Transportation Noise and Vibration—The New Millen-*

- nium, *Proceedings of the Annual Conference of the Australian Acoustical Society*, Gold Coast, Australia, 3–5 November, edited by M. J. Mee, R. J. Hooker, and I. D. M. Hillock (Australian Acoustical Society, Castlemaine, VIC, Australia), pp. 555–560.
- Dupree, R. B., **1981**, “Catalog of STC and IIC ratings for wall and floor/ceiling assemblies—With TL and ISPL data plots,” Office of Noise Control, California Department of Health Services, Berkeley, CA.
- Gradshteyn, I. S., and Ryzhik, I. M., **1965**, in *Table of Integrals, Series, and Products*, prepared by Yu. V. Geronimus and M. Yu. Tseytlin, translated from Russian by Scripta Technica Inc., 4th ed., edited by A. Jeffrey (Academic, New York).
- Heckl, M., and Donner, U. [(**1985**). “Schalldämmung dicker Wände (Sound insulation of thick walls),” *Rundfunktech Mitt.* **29**, 287–291.
- ISO, **2005**, “ISO 15712-1:2005(E) Building acoustics—Estimation of acoustic performance of buildings from the performance of elements—Part 1: Airborne sound insulation between rooms,” International Organisation for Standardization, Geneva, Switzerland.
- Leppington, F. G., Heron, K. H., Broadbent, E. G., and Mead, S. M. (**1987**). “Resonant and nonresonant acoustic properties of elastic panels. II. The transmission problem,” *Proc. R. Soc. London, Ser. A* **412**, 309–337.
- Ljunggren, S. (**1991**). “Airborne sound insulation of thick walls,” *J. Acoust. Soc. Am.* **89**, 2338–2345.
- Magrab, E. B. (**1979**). *Vibrations of Elastic Structural Members* (Sijthoff & Noordhoff, Alphen aan der Rijn, The Netherlands).
- Northwood, T. D. (**1968**). “Transmission loss of plasterboard walls,” Building Research Note, BRN-66, Division of Building Research, National Research Council of Canada, Ottawa, ON, Canada.
- Quirt, J. D. (**1981**). “Measurements of the sound transmission loss of windows,” Building Research Note, BRN-172, Division of Building Research, National Research Council of Canada, Ottawa, ON, Canada.
- Quirt, J. D. (**1982**). “Sound transmission through windows I. Single and double glazing,” *J. Acoust. Soc. Am.* **72**, 834–844.
- Rindel, J. H. (**1975**). “Transmission of traffic noise through windows—Influence of incident angle on sound insulation in theory and experiment,” Technical University of Denmark Report No. 9, The Acoustics Laboratory, Technical University of Denmark, Lyngby, Denmark.
- Sato, H.. (**1973**). “On the mechanism of outdoor noise transmission through walls and windows—A modification of infinite wall theory with respect to radiation of transmitted wave,” *J. Acoust. Soc. Jpn.* **29**, 509–516.
- Sewell, E. C. (**1970**). “Transmission of reverberant sound through a single-leaf partition surrounded by an infinite rigid baffle,” *J. Sound Vib.* **12**, 21–32.
- Villot, M., Guigou, C., and Gagliardini, L. (**2001**). “Predicting the acoustical radiation of finite size multi-layered structures by applying spatial windowing on infinite structures,” *J. Sound Vib.* **245**, 433–455.